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VARIANCE AND BIAS OF CATCH ALLOCATIONS THAT USE THE AGE COMPOSITION OF ESCAPEMENTS

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PREFACE

This report probes the mathematical accuracy and precision of currently used methods of allocating salmon catches to runs of origin with knowledge of the age composition of catches and escapements. Some readers will want to follow the logic behind the mathematics in this report and (hopefully) arrive at the stated conclusions. The derivations are written for these hardy souls. If the text seems overburdened with mathematical symbols, take solace, the equations are more intimidating than complex.

However, most readers will care less about the "means" than about the "ends". If you are interested in the knowledge that bias and variances exist for allocations but not in how they are calculated, I suggest first reading the Abstract, then Sections 1, 2, 5, and finally Section 6, the Discussion. Note that allocations have variances and that these allocations can have bias (variances are a measure of precision, bias is a measure of accuracy and precision). Note also under what conditions biases can occur in allocations. Finally, if after being forewarned about the limitations of the methods you still wish to employ these procedures, use the Fortran subroutines in Appendix A or the Equations 3.20-3 to estimate run size, catch allocations, and their variances.

D.R.B.

ABSTRACT

Information from stratified sampling programs (catch by age; escapement by age and river), counting sites (escapements), and fish tickets (catches) is combined in the Pooled and the Difference methods of allocating catches according to age compositions in escapements. Equations to calculate the variances of catch allocations of and returns to salmon fisheries and Fortran subroutines (for the Pooled method only) to calculate variances are provided. For the Pooled method, exploitation rates among runs within age groups are presumed the same; for the Difference method, exploitation rates within runs among age groups are presumed the same. When exploitation rates on different runs are not the same, the Pooled method: 1) allocates too few fish to runs with higher exploitation rates and too many to runs that experience smaller rates within an age group, 2) produces large relative biases in allocations to small runs when relative bias in allocations to large runs is small within an age group, and 3) produces relative biases in each allocation within an allocation scheme that never completely cancel within the scheme. When exploitation rates on different age classes are not the same, the Difference method allocates too many fish to age groups that are large and less exploited at the expense of all other age groups and runs. Imprecise estimates from inadequate catch and escapement sampling programs adversely affect the accuracy of both methods, but affect the accuracy of allocations from the Difference method most. The Pooled method is identified as the better method because it is less sensitive to differences in exploitation rates. Circumstances under which age composition methods can be used with negligible bias are listed. Catches from hypothetical and real fisheries were allocated with both methods as examples.

KEY WORDS: Catch allocation, Pooled method, Difference method, bias, variance, error, salmon return calculation

1. INTRODUCTION

The Difference and Pooled methods are tools to allocate salmon catches to runs with information on age composition of catch and escapement. Both methods require estimates of age composition from stratified random, stratified systematic, or multistage sampling program (see Cochran 1977, Chapter 10 for details). Also, the age composition of the escapement is assumed representative of the age composition of the catch for both methods. The Difference method uses differences between the age composition of the escapements to allocate catch (Seibel 1972). The Pooled method uses the run composition of the escapements pooled for an age group.

Both the Pooled and the Difference methods are popular because they require only simple statistics that are gathered in the normal course of monitoring a salmon fishery. The Pooled method is now used in the Bristol Bay sockeye salmon *(Oncorhynchus nerka)* fishery and in other fisheries in Alaska when other means of allocation, such as scale pattern analysis, are unavailable or unworkable. The Difference method has been used largely for pink salmon *(O. gorbuschua)* with attributes other than age (Worlund and Fredin 1962) and for Japanese high seas gill net fishery for sockeye salmon (Fredin and Worlund 1974).

Although both methods are easy to understand and use, the variances of their allocations and their sensitivity to violations of the assumptions upon which they are based have not been thoroughly investigated. If managers are to have confidence in these techniques, variances and biases must be known for each method.

2. THE ESTIMATORS

Because both estimators are too cumbersome to describe in single equations, they are each decomposed into several equations and are programmed in Fortran subroutines listed in Appendix A. Appendix B contains the definitions of all the notation in the equations and in the rest of the report. Some general definitions and conventions are:

1) The letter C represents a catch, E an escapement, P an age as a proportion of an escapement, and Q an age as a proportion of the catch.

2) The symbol ^ above a letter means an estimated value is represented.

3) The subscripts i and j correspond to age and river, respectively, and k and h indicate the sampling stratum in the catch and in the escapement sampling programs, respectively. Subscripts s and t refer to rivers other than river j.

4) Subscripts have positions from left to right i, j, (h or k).

5) Any $\cdot$ in place of a subscript indicates that the variable is summed over that subscript; for instance, $C_{i.}$ is the catch of fish of age i, $C_{.j}$ is the catch of fish from river j, C_{ij} is the catch of fish of age i from river j, and $C_{..k}$ is the catch during period k. The two exceptions are total catch (C) and total return (R) which have no dots at all.

6) Capitals of subscripts represent the maximum values that the subscripts can attain.

Because both methods require information from catch and escapement sampling programs for age composition, the description of the first stages of each method are the same. Imagine a fishery with I ages and J rivers, a catch sampling program with K strata, and an escapement sampling program in river J with H_j strata:

$$2.1)\quad c_{i.} = c\hat{Q}_i$$

$$2.2)\quad \hat{Q}_i = \sum_{k=1}^{K} \hat{Q}_{ik} \frac{C_{..k}}{C}$$

$$2.3)\quad \hat{E}_{ij} = \hat{E}_{.j}\hat{P}_{ij}$$

$$2.4)\quad \hat{P}_{ij} = \frac{1}{E_{.j.}} \sum_{h=1}^{H_j} \hat{P}_{ijh}E_{.jh}$$

To use the methods to make allocations of the catch by river:

1) First, estimate the proportion of the population of each age with stratified or multistage sampling programs on the catch $(\hat{Q}_{ik}, \hat{Q}_i)$ and on each escapement $(\hat{P}_{ijh}, \hat{P}_{ij})$ for each river. The estimated proportion is the weighted sum of the proportions for each stratum. A weight is the ratio of the number caught (or escaped by river) during the sampling period to the number caught (or escaped by river) for the season (Eq. 2.2 or 2.4).

2) Then estimate the numbers by age for the season for each escapement and for the catch $(\hat{E}_{ij}, \hat{C}_i)$. Numbers by age are given by the product by the estimated proportion by age for the catch or for the escapement by river and the numbers caught or the numbers escaped by river (Eq. 2.1 or 2.3).

Pooled Method

The Pooled method requires the assumption that within each age group, the percentage of all escapement that went to river j is the percentage of fish from that river in the catch:

$$2.5)\quad \hat{C}_{ij} = \frac{\hat{C}_{i.}\hat{E}_{ij}}{\hat{E}_{i.}}$$

Eq. 2.5 provides one allocation of fish of age i to river j. Eq. 2.6 represents the entire allocation scheme for a fishery.

2.6) $C = D[C_i]\theta$

where C is an IxJ matrix of c_{ij}, $D[C_i]$ is an IxI diagonal matrix of the catches by age, and θ is an IxJ matrix of θ_{ij} $(= E_{ij}/E_i)$. There is only one possible allocation scheme, and it is composed of IxJ separate allocations. Because allocations are made according to proportions within an age, the effect of gear selectivity on escapement age compositions is irrelevant unless runs have significantly differently sized fish of the same age. The only assumption is that $c_{ij}/c_{i\cdot} = E_{ij}/E_{i\cdot}$. The number of runs, ages, and the differences in age compositions among runs have no bearing on the existence of an allocation scheme from the Pooled method.

Difference Method

The Difference method requires the assumption that some difference exists between the age compositions of the runs. The number of fish of age i in the catch is the sum of the products of catch by run and the proportion of that run of a given age. Since the age composition of the catch of a single run is unknown, the age composition of its escapement is used as an estimate. From Eq. 5 in Worlund and Fredin (1962), the Difference method is a series of simultaneous equations, one equation for each age group in the fishery:

2.7)
$$\begin{array}{ccccccccc}
p_{11}c_{\cdot 1} & + & p_{12}c_{\cdot 2} & + & \dots & + & p_{1J}c_{\cdot J} & = & c_{1\cdot} \\
p_{21}c_{\cdot 1} & + & p_{22}c_{\cdot 2} & + & \dots & + & p_{2J}c_{\cdot J} & = & c_{2\cdot} \\
\cdot & & \cdot & & & & \cdot & & \cdot \\
\cdot & & \cdot & & & & \cdot & & \cdot \\
\cdot & & \cdot & & & & \cdot & & \cdot \\
p_{I1}c_{\cdot 1} & + & p_{I2}c_{\cdot 2} & + & \dots & + & p_{IJ}c_{\cdot J} & = & c_{I\cdot}
\end{array}$$

There are several ways to solve the simultaneous equations in Eq. 2.7 to get an allocation scheme, depending on how many ages and runs are involved. If the number of ages is the same as the number of runs $(I = J)$, only one allocation scheme can be calculated; if there are more runs than ages $(I < J)$, no scheme is possible; and if there are more ages than runs $(I > J)$, more than one scheme can be calculated. The number of allocation schemes that can be made is the combination of I ages taken J at a time $\{= I!/[J!(I-J)!]\}$. To solve a set of the simultaneous equations in Eq. 2.7:

2.8) $C_j = P^{-1}C_i$

where C_j is a $Jx1$ column vector with elements $c_{\cdot 1}, c_{\cdot 2}, \dots, c_{\cdot J}$, C_i is a $Jx1$ column vector with elements that are catches by specific (but not necessarily all) ages, and P is a JxJ matrix with elements p_{ij} . For the special case of two ages $(I=2)$ and two runs $(J=2)$, Worlund and Fredin (1962) use constraints $[p_{21} = 1 - p_{11}$ and $c_{\cdot 2} = c - c_{\cdot 1}]$ to produce an allocation scheme from Eq. 2.7:

$$2.9)\quad \hat{c}_{.j}/C = \frac{\hat{Q}_i - \hat{P}_{it}}{\hat{P}_{ij} - \hat{P}_{it}}$$

where $\hat{c}_{.j}/C$ is the proportion of the catch that is from run j, and $j \neq t$. This special case is an extension of Eq. C.1 (Appendix C).

When the number of ages is greater than the number of runs, more than one allocation scheme is possible, so which is the best to use? Worlund and Fredin (1962) suggest using least-squares techniques to solve Eq. 2.7 to get the one, best allocation scheme. The $\hat{c}_{.j}$ are the estimated slope parameters, and the intercept must be zero. When the dependent variables $(\hat{c}_{i.})$ have different variances, a weighted regression should be used. Because the independent variables have variances, a functional regression is needed, and because the numbers of ages in the regression is probably small, a GM regression will have to suffice for the AM regression (Ricker 1975, p. 351-2). Seibel (1972) suggests using multiple allocation schemes to adjust the Difference method for the effect of the fishery on age compositions in escapements. Because gear selectivity will cause the age composition of the escapements to deviate from that of the run, multiple allocation schemes can be used to correct for this bias.

The solution to C_j is still not an allocation scheme. No matter what variant of the Difference method is used, each estimate of catch by run $(\hat{c}_{.j})$ must be split into catch by age by run $(\hat{c}_{ij})$ through estimates of the age and run composition of its escapement before an allocation scheme is obtained.

No matter what the procedure used to solve Eq. 2.7, no set of two or more runs can have the same age composition (i.e., two or more columns in P may not be the same) if allocations are to be made to all runs. If age compositions are the same for two or more runs, these runs must be treated as a single group to use the Difference method.

3. VARIANCE OF THE ESTIMATORS

Although sampling programs do not affect the estimator, they do affect the estimates. Allocations based on "known" age compositions have no variance; if every fish caught in a run is aged, both methods provide allocation schemes with perfect precision (but not necessarily with perfect accuracy). But because aging all fish is not feasible, catches and escapements are sampled and each sample has a variance. The variances of allocations become expansions of variances according to the equations which define each estimator.

Even more so than the estimators, their variance is too cumbersome to describe in a single equation, so the variance equations are listed in several equations. And like the estimator, the variance equations for the Pooled method are coded in Fortran subroutines listed in Appendix A.

Catch

The variance of estimated catch by age is the sum over time of the products of the estimated age composition weighted by the period is catch (see Cochran 1977, p.

107-9). Because the catch for a period is known from fish ticket information, not estimated, the variance of the estimated catch by age is the product of the variance of the estimated age composition for the season $V[\hat{Q}_i]$ and the square of the season's catch C[1]. The catch-squared terms cancel in the equation to give:

$$3.1)\quad V[\hat{C}_{i.}] = \sum_{k=1}^{K} \frac{C^2_{..k}(C_{..k} - N_k)\hat{Q}_{ik}(1 - \hat{Q}_{ik})}{(C_{..k} - 1)(N_k - 1)}$$

$$3.2)\quad V[\hat{Q}_i] = V[\hat{C}_i]/C^2$$

The ratio $\{C_{..k} - N_k)/(C_{..k} - 1)\}$ is the finite population correlation factor which reduces sample variance as the sample size approaches population size[2]. A stratified sampling program is assumed in Eq. 3.1-2. If a multistage sampling program is used instead, Cochran (1977, Chapter 10) lists the appropriate equations.

Known Escapement

When escapement is known, not estimated, $E_{.jh}$ is not a variable, but a constant. Sonar, towers, and weirs provide estimates of escapement that are constants when total counts are made. With escapement a constant, the variance of escapement by river by age is:

$$3.3)\quad V[\hat{E}_{ij}] = \sum_{h=1}^{H_j} \frac{E^2_{.jh}(E_{.jh} - N_{jh})\hat{P}_{ijh}(1 - \hat{P}_{ijh})}{(E_{.jh}-1)(N_{jh} - 1)}$$

$$3.4)\quad V[\hat{P}_{ij}] = V[\hat{E}_{ij}]/E_{.j}^2$$

where $\hat{P}_{ijh}$ is the proportion of the escapement passing by the sonar, tower, or weir on river j during period h that is comprised of age group i. Estimates of $\hat{P}_{ijh}$ come from stratified sampling programs for the age composition of the escapement. The constant $E_{.jh}$ is the escapement to river j that pass the counters during period h, and N_{jh} is the number of fish by age group captured in the sampling program during period h. The term $\{(E_{.jh} - N_{jh})/(E_{.jh} - 1)\}$ is the finite population correction factor.

The variance of $\hat{E}_{i.}$ is the sum of variances of the $\hat{E}_{ij}$ over rivers. From Mendenhall et al. (1981, p. 207), the variance of a sum is:

$$3.5)\quad V[A_1 + A_2 + \ldots A_n] = V[A_1] + V[A_2] + \ldots V[A_n] + \text{Covariances}$$

---

[1] If catch is estimated and has a variance, Eq. 3.1 will look more like Eq. 3.8 provided that the variance of the estimated catch is known.

[2] A catch (or an escapement) in which all fish are aged has a known age composition with no variance.

In this derivation, the covariance terms in the above equation can be dropped because samples of age composition and escapement come from different periods and are independently drawn. Thus,

$$3.6)\quad V[\hat{E}_{i.}] = \sum_{j=1}^{J} V[\hat{E}_{ij}] \quad .$$

Estimated Escapement

When escapements are estimated (for instance, by expanding incomplete counts from weirs, sonar, and towers), the variances of escapement by age by river estimated with Eq. 3.4 are biased (see Cochran 1977, p. 117-119). Instead of a product of a constant and a variable, $\hat{E}_{ij}$ is the product of two variables. From Goodman (1960, Eq. 2), the variance of a product of two independent variables is:

$$3.7)\quad V[AB] = A^2 V[B] + B^2 V[A] - V[A]V[B] \quad .$$

For $V[\hat{E}_{ij}]$, $V[\hat{E}_{ijh}]$ is calculated first:

$$3.8)\quad V[\hat{E}_{ijh}] = \hat{E}_{\cdot jh}{}^2 V[\hat{P}_{ijh}]F_a + \hat{P}_{ijh}{}^2 V[\hat{E}_{\cdot jh}] - V[\hat{E}_{\cdot jh}]V[\hat{P}_{ijh}] \quad ,$$

$$3.9)\quad V[\hat{P}_{ijh}] = \frac{\hat{P}_{ijh}(1-\hat{P}_{ijh})}{N_{jh}-1} \quad ,$$

where the finite population correction factor for the age composition is $F_a = (\hat{E}_{\cdot jh} - N_{jh})/(\hat{E}_{\cdot jh} - 1)$. Unlike Eq. 3.3, the estimated escapement by river by stratum in Eq. 3.8 has a variance. Becker (1962) gives equations to calculate $\hat{E}_{\cdot jh}$ and $V[\hat{E}_{\cdot jh}]$ from expanded tower counts; variances for escapements based on expanded weir and sonar counts are calculated in the same fashion. Because finite population correction factors for escapement sampling are included in Becker's equations, these correction factors are not needed in Eq. 3.8. To get $V[\hat{E}_{ij}]$, the variances by period must be summed over periods:

$$3.10)\quad V[\hat{E}_{ij}] = \sum_{h=1}^{H_j} V[\hat{E}_{ijh}] \quad ,$$

The covariance terms from Eq. 3.5 are omitted because the estimates of age composition and escapement are made with data from two, independently operated sampling programs and are therefore independent. The variance of $\hat{E}_{i\cdot}$ is found as in Eq. 3.6.

An alternative method to calculate $V[\hat{E}_{ij}]$ is to ignore the bias and use Eq. 3.3. The bias is negligible as long as the partial escapement counts are much larger than the sample sizes to determine age proportion. This is often the case -- the precision of the escapement estimates is much better than that for the estimates of age composition. If estimates of $V[P_{ij}]$ are needed (as for the Difference method) when escapements are estimated, Eq. 3.4 is a good approximation under these circumstances.

Allocation with the Pooled Method

When the variances of the three components of Eq. 2.5 $\{\hat{C}_{i\cdot}, E_{ij}, \text{ and } \hat{E}_{i\cdot}\}$ are known, the next is to separate Eq. 2.1 into a product of a variable $\hat{C}_{i\cdot}$ and a ratio $\hat{E}_{ij}/\hat{E}_{i\cdot}$.

The variance of a ratio of two variables can be approximated through a Taylor series expansion of the ratio {from Seber (1973, p. 8)}:

$$3.11)\quad V[g(\mathrm{X})] \cong \sum_s V[x_s]\left(\frac{\partial g}{\partial x_s}\right)^2 + 2\sum_s \sum_{<t} Cov[x_s x_t]\frac{\partial g \partial g}{\partial x_s \partial x_t},$$

where g is a function of X and X is a vector of variables. If g is a function of variables A and B where $g(A,B) = A/B$, then

$$3.12)\quad V[A/B] = \left[\frac{A}{B}\right]^2\left[\frac{V[A]}{A^2} + \frac{V[B]}{B^2} - \frac{2Cov[A,B]}{AB}\right].$$

Replacing A and B with $\hat{E}_{ij}$ and $\hat{E}_{i\cdot}$,

$$3.13)\quad V[\hat{E}_{ij}/\hat{E}_{i\cdot}] = \left[\frac{\hat{E}_{ij}}{\hat{E}_{i\cdot}}\right]^2\left[\frac{V[\hat{E}_{ij}]}{\hat{E}_{ij}^{\,2}} + \frac{V[\hat{E}_{i\cdot}]}{\hat{E}_{i\cdot}^{\,2}} - \frac{2Cov[\hat{E}_{ij},\hat{E}_{i\cdot}]}{\hat{E}_{ij}\hat{E}_{i\cdot}}\right].$$

Unlike in the previous equations, the covariance term in Eq. 3.13 can't be ignored. The variables $\hat{E}_{ij}$ and $\hat{E}_{i\cdot}$ are not independent because the former is one of the elements of the latter. However, the covariance of $\hat{E}_{ij}$ and $\hat{E}_{i\cdot}$ is equal to $V[\hat{E}_{ij}]$. The covariance of two sums of variables is the sum of the covariances of all possible combinations of the elements of the two sums (Mendenhall et al. 1981, p. 207). Consider $\hat{E}_{ij}$ a sum of one element and $\hat{E}_{i\cdot}$ a sum of J elements. The escapement $\hat{E}_{ij}$ from one river is estimated independently from all other rivers and is therefore independent of all elements in $\hat{E}_{i\cdot}$ except one. That one exception is itself. But $Cov[\hat{E}_{ij},\hat{E}_{ij}] = V[\hat{E}_{ij}]$. Because of their independence, the covariances between $\hat{E}_{ij}$ and all other elements in $\hat{E}_{i\cdot}$ is zero. Simplifying Eq. 3.13,

$$3.14)\quad V[\hat{E}_{ij}/\hat{E}_{i\cdot}] = \left[\frac{\hat{E}_{ij}}{\hat{E}_{i\cdot}}\right]^2\left[\frac{V[\hat{E}_{ij}}{\hat{E}_{ij}^{\,2}} + \frac{V[\hat{E}_{i\cdot}]}{\hat{E}_{i\cdot}^{\,2}} - \frac{2V[\hat{E}_{ij}]}{\hat{E}_{ij}\hat{E}_{i\cdot}}\right].$$

Now the components of the product can be multiplied according to Eq. 3.7 to give the final, desired result:

$$3.15)\quad V[\hat{C}_{ij}] = \hat{C}_{i\cdot}^{\,2}V[\hat{E}_{ij}/\hat{E}_{i\cdot}] + \{\hat{E}_{ij}/\hat{E}_{i\cdot}\}^2 V[\hat{C}_{i\cdot}] - V[\hat{C}_{i\cdot}]V[\hat{E}_{ij}/\hat{E}_{i\cdot}]$$

Because the catch sampling and escapement sampling programs are conducted independently, catch by age $(\hat{C}_{i\cdot})$ and the ratio $\hat{E}_{ij}/\hat{E}_{i\cdot}$ are independent as required by Eq. 3.7.

Return with the Pooled Method

Once an allocation is made, the run size by age and river $(\hat{R}_{ij})$ is:

$$3.16)\quad \hat{R}_{ij} = \hat{E}_{ij} + \hat{C}_{ij} = \hat{E}_{ij} + \hat{C}_{i.}\hat{E}_{ij}/\hat{E}_{i.} = \hat{E}_{ij}\{1 + \hat{C}_{i.}/\hat{E}_{i.}\}$$

The estimate $\hat{E}_{ij}$ is again twice used which makes covariance between $\hat{E}_{ij}$ and $\hat{C}_{ij}$ obvious. Unfortunately, $\hat{E}_{ij}$ is both multiplied and added in Eq. 3.16 which precludes a straightforward solution to $V[\hat{R}_{ij}]$. But fortunately, calculation of an approximate variance for $\hat{R}_{ij}$ is possible with Eq. 3.11. In this situation, g is Eq. 3.16 and

$$3.17)\quad V[\hat{R}_{ij}] \cong \hat{A}^2V[\hat{C}_{i.}] + \{1 + \hat{B} - \hat{A}\hat{B}\}^2V[\hat{E}_{ij}] + \hat{A}^2\hat{B}^2\{V[\hat{E}_{i.}] - V[\hat{E}_{ij}]\}$$

where $\hat{A} = \hat{E}_{ij}/\hat{E}_{i.}$ and $\hat{B} = \hat{C}_{i.}/\hat{E}_{i.}$ Because all $\hat{E}_{ij}$ and $\hat{C}_{i.}$ are estimated with independent sampling programs, no covariance are needed in Eq. 3.17. Like the variance for the allocation, Eq. 3.17 is encoded in a Fortran subroutine listed in Appendix A.

Allocation and Return with the Difference Method

How variances are calculated for an allocation made with the Difference method depends on the number of runs and the number of ages. If there are more ages than runs $(I > J)$, least-squares techniques can be used to estimate the catch by run, and the variance of the parameter estimate is the variance of catch by run $V[\hat{C}_{.j}]$. An allocation for age i in run j is the product of catch by run and the proportion of escapement j that is age i $(\hat{C}_{.j}$ by $\hat{P}_{ij})$. The approximate variance of an allocation from Eq. 3.11 is:

$$3.18)\quad V[\hat{C}_{ij}] \cong \hat{P}_{ij}^2V[\hat{C}_{.j}] + \hat{C}_{.j}^2V[\hat{P}_{ij}] + 2\hat{P}_{ij}\hat{C}_{.j}Cov[\hat{P}_{ij},\hat{C}_{ij}] \quad .$$

The approximate variance of the return (catch plus escapement) is

$$3.19)\quad V[\hat{R}_{ij}] = V[\hat{C}_{ij}] + V[\hat{E}_{ij}] + 2Cov[\hat{C}_{ij},\hat{E}_{ij}] \quad .$$

Because $\hat{P}_{ij}$ is used to calculate both $\hat{C}_{.j}$ and $\hat{E}_{ij}$, the covariance terms in Eq. 3.18-9 are not zero. Least-square techniques provide estimates of covariance among parameters (among the $\hat{C}_{.j}$) and among independent variables (among the $\hat{P}_{ij}$), but not among parameters and independent variables, and without this last set of covariances, precise variances for allocations from the Difference method are not attainable.

How to calculate the variances of the allocations when $I=J$ depends on the rank of P. When the rank of two $(I = J = 2)$, multiply Eq. 2.8 by the catch and by the proportion of run j made up of age i:

3.20) $\hat{C}_{ij} = C\hat{P}_{ij} \dfrac{\hat{Q}_i - \hat{P}_{it}}{\hat{P}_{ij} - \hat{P}_{it}}$

The approximate variance of an allocation $\hat{C}_{ij}$ is (see Eq. 3.11)

3.21) $V[\hat{C}_{ij}] = \dfrac{1}{(\hat{P}_{ij} - \hat{P}_{it})^2} \left[C^2\hat{P}_{ij}^{\ 2} V[\hat{Q}_i] + \hat{P}_{ij}^{\ 2}\hat{C}_{.t}^{\ 2} V[\hat{P}_{it}] + \hat{C}_{ij}^{\ 2} V[\hat{P}_{ij}] \right]$.

Because all the variables in Eq. 3.20 are estimated with independent sampling programs, there are no covariances in Eq. 3.21. The variance of the return for the Difference method is based on the sum of catch by age and run $\hat{C}_{ij}$ (Eq. 3.21) and the escapement of age i to run j, $\hat{E}_{ij}$ (Eq. 2.3), or

3.22) $\hat{R}_{ij} = C\hat{P}_{ij} \dfrac{\hat{Q}_i - \hat{P}_{it}}{\hat{P}_{ij} - \hat{P}_{it}} + \hat{E}_{.j}\hat{P}_{ij}$

The approximate variance of the return $\hat{R}_{ij}$ (see Eq. 3.11) is

3.23) $V[\hat{R}_{ij}] = V[\hat{C}_{ij}] + V[\hat{P}_{ij}] \left[\dfrac{2\hat{C}_{ij}\hat{E}_{.j}}{\hat{P}_{ij} - \hat{P}_{it}} + \hat{E}_{.j}^{\ 2} \right] + \hat{P}_{ij}^{\ 2} V[\hat{E}_{.j}]$

When the escapement is known by complete counts, the last term in Eq. 3.23 $\{V[\hat{E}_{.j}]\hat{P}_{ij}^{\ 2}\}$ is zero. Because all the variables in Eq. 3.22 are estimated with independent sampling programs, there are no covariances in Eq. 3.23.

Pella and Robertson (1979) developed a method to calculate the variances of allocations (allocations as fractions) made with scale pattern analysis on fisheries with more than two runs; this technique is directly applicable to allocations made with the Difference method. The correction matrix is P, and the vector of proportions of the attribute in the mixture is $\mathrm{C_i}$. However, Cook (1982) used Monte Carlo simulation to show that the estimated variances using Pella and Robertson's methods are conservative (i.e., too large). Also, the variances from Pella and Robertson (1979) are for the fraction of the catch composed of fish from run j and not variances of the allocation or of the return.

When the rank of P is more than two $(I = J > 2)$, Monte Carlo simulation is the best procedure by which to calculate variances for an allocation scheme. Each element of the general solution to the Difference method ($\hat{Q}_i$ and all $\hat{P}_{ij}$; see Eq. 2.8) has an estimated sampling variance. Random numbers, each distributed uniformly between 0 and 1, are generated and transformed with a computer into values for $\hat{Q}_i$ and $\hat{P}_{ij}$ that are distributed according to their binomial distributions with variances $V[\hat{Q}_i]$ and $V[\hat{P}_{ij}]$. Each set of P and Q are inserted into Eq. 2.8, and a set of C and R are generated. If this process is repeated many times, a population of R is generated from which a variance can be calculated directly.

4. ASSUMPTIONS AND BIASES

Although aging every fish in the escapement and in the catch produces allocations with perfect precision (no variance), the estimator could still produce inaccurate allocations if the assumptions upon which the estimator rests are violated.

Imprecise and non-representative samples of age composition will produce imprecise and inaccurate estimates of $C_{i.}$, E_{ij}, and subsequently C_{ij} . This problem is best solved with good sampling programs for estimating age compositions. When age compositions change with time and random access to the population produces representative samples, a stratified or a systematic sampling design is adequate to produce good estimates. However, if the age composition of the catch varies considerably with the manner in which the catch is landed, a multistage sampling program should be used.

Another bias occurs when not all the runs are included in the allocation. If the escapements of some of the runs are not included in the allocation, the remaining runs will be allocated too many fish. And if some of the escapements are not included in the calculation of $\hat{E}_{i.}$, the allocation will be biased.

Pooled Method

Another bias in a catch allocation made with the Pooled method occurs when the run composition of the escapement no longer reflects the run composition of the catch. This happens when different runs within the same age group experience different exploitation rates.

Partitioning the return by age and by run is the first step in isolating the effects of exploitation rates on the accuracy of the estimates from the Pooled method:

$$4.1) \quad R_{ij} = Rp_i r_{ij} \quad ,$$

where R_{ij} is the return of salmon of age i to river j, R is the return in a year of all ages going to all rivers, p_i is the proportion of the return of age i, and r_{ij} is the proportion of the return of age i that goes to river j $(r_{i.} = 1)$[1]. The catch by age and river is the product of Eq. 4.1 and the annual exploitation rate by age and river (U_{ij}):

$$4.2) \quad C_{ij} = Rp_i r_{ij} U_{ij} \quad .$$

Both sides of Eq. 4.2 can be summed to obtain catch by age and the exploitation rate by age (U_i):

---

[1] The sum of the proportion p_i over i (ages) is one; the sum of the proportion r_{ij} over j is one.

$$4.3) \quad C_{i\cdot} = \sum_{j=1}^{J} Rp_i r_{ij} U_{ij} = Rp_i \sum_{j=1}^{J} r_{ij} U_{ij} = Rp_i U_i \quad .$$

Note that U_i is a weighted sum of the U_{ij}, the weights being the proportion of the age group from river j. The escapement by age and river is the complement of the catch by age and rivers,

$$4.4) \quad E_{ij} = Rp_i r_{ij}(1 - U_{ij}) \quad .$$

If the right-hand sides of Eq. 4.3 and 4.4 are substituted for equivalent terms in Eq. 2.5, the new version of the estimator becomes

$$4.5) \quad \hat{C}_{ij} = \frac{Rp_i U_i Rp_i r_{ij}(1 - U_{ij})}{\sum_{j=1}^{J} Rp_i r_{ij}(1 - U_{ij})} = \frac{Rp_i r_{ij}(1 - U_{ij})U_i}{(1 - U_i)} \quad ,$$

since $U_i = \sum_{j=1}^{J} r_{ij} U_{ij}$ and $\sum_{j=1}^{J} r_{ij} = 1$. For no bias to occur, Eq. 4.5 must equal Eq. 4.2:

$$4.6) \quad \hat{C}_{ij} = C_{ij} = \frac{Rp_i r_{ij}(1 - U_{ij})U_i}{(1 - U_i)} = Rp_i r_{ij} U_{ij} \quad .$$

When the product $Rp_i r_{ij}$ is removed from both sides of Eq. 4.6,

$$4.7) \quad \frac{(1 - U_{ij})U_i}{(1 - U_i)} = U_{ij} \quad .$$

For Eq. 4.7 to be true,

$$4.8) \quad U_{ij} = U_i = \sum_{j=1}^{J} r_{ij} U_{ij} \quad .$$

How much bias will be estimator impart to the allocation scheme if the U_{ij} are not equal? Bias is defined as the relative deviation in the estimated allocation of catch by age by river from the actual value,

$$4.9) \quad B_{ij} = \frac{\hat{C}_{ij} - C_{ij}}{C_{ij}} = \frac{\hat{C}_{ij}}{C_{ij}} - 1 \quad .$$

If substitutions in Eq. 4.9 for C_{ij} are made from Eq. 4.2 and for $\hat{C}_{ij}$ are made from Eq. 4.5,

$$4.10) \quad B_{ij} = \frac{Rp_i r_{ij}(1 - U_{ij})U_i}{(1 - U_i)Rp_i r_{ij} U_{ij}} - 1 \quad ,$$

assuming that $U_{ij} > 0$. The product $Rp_i r_{ij}$ can be removed from, and weights added to, Eq. 4.10, giving

$$4.11) \quad B_{ij} = \frac{(1 - w_{ij}U)U_i}{(1 - U_i)w_{ij}U} - 1 = \frac{U_i}{[Uw_{ij}][(1 - U_i)]} - \frac{1}{[(1 - U_i)]} \quad .$$

The product $w_{ij}U = U_{ij}$ where w_{ij} is a weight describing deviations in U_{ij} from U, a standard variation exploitation rate. Consider the w_{ij} as an element in a vector W_i such that $W_i = [1,1,1 \ldots 1]$ when there is no bias in the allocation scheme. As individual U_{ij} deviate from U, the corresponding w_{ij} is less than or greater than one (for all w_{ij}, $0 < w_{ij} \leqslant 1/U$). No weights are placed on the U_i because these exploitation rates are estimated with catch and escapement sampling programs for age composition, and act as constraints on the biases in the allocation scheme. An estimated U_{ij} can not be calculated solely from sampling programs for age composition. Because an estimate of U_i is available:

$$4.12) \quad B_{ij} = c\frac{[\ \hat{U}_i\ ]}{[Uw_{ij}]} - c$$

where $c = 1/(1-\hat{U}_i) = \hat{R}_{i.}/\hat{E}_{i.}$. From Eq. 4.3 and 4.8,

$$4.13) \quad U_i = U \sum_{j=1}^{J} r_{ij}w_{ij} \quad .$$

Putting Eq. 4.13 into Eq. 4.12 gives

$$4.14) \quad B_{ij} = \frac{c \sum_{t=1}^{J} r_{it}w_{it}}{w_{ij}} - c$$

and B_{ij} drops to zero when all w_{ij} equal one. If only $w_{ij} \neq 1$,

$$4.15) \quad B_{ij} = - \frac{c(1 - r_{ij})(w_{ij} - 1)}{w_{ij}}$$

and if $w_{it} \neq 1$ while $w_{ij} = 1$,

$$4.16) \quad B_{ij} = cr_{it}(w_{it} - 1) \quad .$$

Note that Eq. 4.14 through 4.16 do not contain U, which makes U a scale factor and B_{ij} independent of U.

Inspection of Eq. 4.14 through 4.16 provides answers to general questions about the nature of the bias in allocations made with the Pooled method and about the relationships among bias, exploitation rates, and run compositions. What happens when only $U_{ij} \neq U$? When U_{ij} and one or more U_{it} do not equal U? Are there conditions where the bias is insensitive to differences in exploitation rates? Can biases cancel one another? Can only one allocation in an allocation scheme be biased?

1) <u>From Eq. 4.15, as a run is exploited harder than other runs, its fish are erroneously allocated to other runs, and when exploited not as hard, it's erroneously allocated fish from the other runs.</u> When $w_{ij} > 0$, $B_{ij} < 0$, and $C_{ij} < C_{ij}$. When $w_{ij} < 0$, $B_{ij} > 0$, and $C_{ij} > C_{ij}$.

2) <u>From Eq. 4.14, the amount under or over allocated is a product of the deviations in exploitation rates and the relative sizes of the runs.</u> If run j is large relative to all runs, a small deviation in its exploitation rate will still move large numbers to the other runs in the allocation; if run j is small relative to other runs, a large deviation in its exploitation rate is needed to produce the same effect.

When one run in a two-run fishery has a biased allocation, the allocation to the other run is biased also. But what about three or more runs with runs exploited above the standard rate, runs exploited at the standard rate, and runs exploited below the standard rate? Can the biases be completely antagonistic? No.

3) <u>From Eq. 4.14, an allocation scheme for a fishery will always be biased when the exploitation rate for one run is different than that of the others.</u> For the biases in each allocation to cancel one another to provide an unbiased allocation scheme, all B_{ij} must be zero, and all Eq. 4.14 must be equal for all j. Since the constant c and the summation in Eq. 4.14 are the same regardless of j, the values of w_{ij} must be equal for all j for all the B_{ij} to be zero. Since U is a scale factor, equal w_{ij} become equal to one by adjusting the scale factor.

So much for answers to general questions; now to the specific questions: how big is the bias when exploitation rates are different among runs? The magnitude of the bias depends on the variation in exploitation rates (the w_{ij}), the exploitation rate for the age group (U_i) and the relative size of the runs involved (the r_{ij}). In Eq. 4.15, B_{ij} is a function of w_{ij}, r_{ij}, and U_i (remember $c = 1/[1-U_i]$). When B_{ij} (Eq. 4.14) is plotted against w_{ij} at several values of the other variables in a two-run fishery, Figure 4.1 results. Line A occurs when 10% of age i in the return is caught and run j has 90% of fish that age. Line B occurs in one of two circumstances: 1) 90% of age group i is caught and run j has 90% of that age group or 2) 10% of age group i is caught and run j has 10% of that age group. Line C occurs when 90% of age group i in the return is caught and run j has 10% of fish that age. A value of 1 for B_{ij} represents a doubling of the correct allocation, a value of 2 represents a tripling, etc.; a value of -1 is the lowest value possible and represents no allocation when one should have been made. Potential for bias is greatest for small, heavily exploited runs and is least for large, lightly exploited runs. The overall exploitation rate for the age group has the greatest effect on

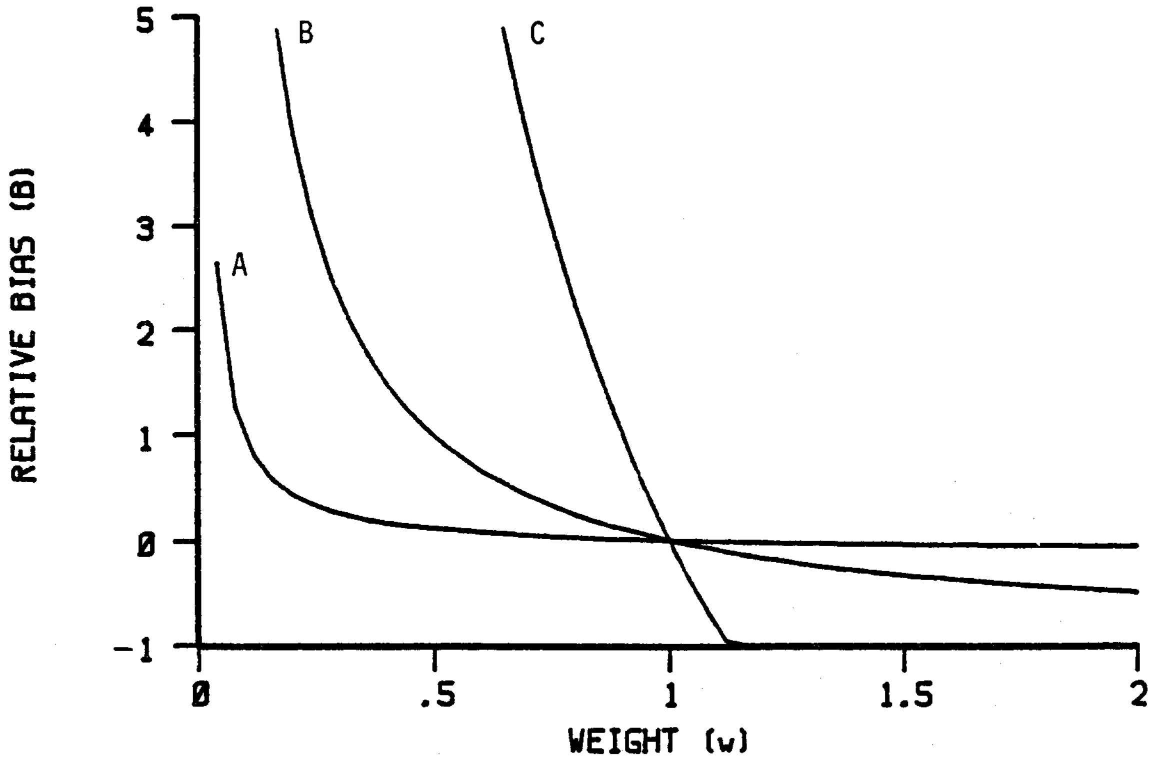

Figure 4.1 Relative bias in estimated catch allocations made with the Pooled method under different exploitation rates and run composition of the catch when the return is composed of two runs. Line A corresponds to large, lightly exploited runs, Line B to small, lightly exploited or large, intensely exploited runs, and Line C to small, intensely exploited runs. See the text for more details.

potential bias (line C). Any bias the fishery imparts because of differential exploitation will be large only when the fishery takes a large portion of the run. No matter how great the differential in exploitation rates among runs, a small catch leaves the run composition of the escapement a fair approximation of the run composition of the return (line A).

Although it is not possible to correct for biases unless the exploitation rates are known, knowledge of the biases for some typical situations is instructive. In Figure 4.2, the top plot corresponds to a two-run fishery in which the observed exploitation rate for age i is 50% and each run corresponds to half the return. The dotted lines connect bias with different weights and their corresponding exploitation rates. Note that when run j is exploited at 45%, the bias in the allocation is 11%; when run j is exploited at 55%, the bias is -9%. The lower plot in Figure 4.2 represents a two-run fishery in which the observed exploitation rate for age i is still 50%, but one run is only a third of the return. Note that when the smaller run is exploited at 45%, the bias in the allocation to that run is 15%, and when the smaller run is exploited at 55%, the bias is -12%.

Difference Method

While the Pooled method presumes equal exploitation rates among runs within an age group, the Difference method presumes equal rates among ages within runs. The Difference method uses a difference among runs in some attribute whose frequency is known for each run. The only estimate of the age composition of each run available is the estimate of the age composition of the escapement. The true frequency of the attribute age in a run is $R_{ij}/R_{.j}$, or

$$4.17)\quad \frac{R_{ij}}{R_{.j}} = \frac{Rp_i r_{ij}}{\sum_{s=1}^{I} Rp_s r_{sj}} = \frac{p_i r_{ij}}{\sum_{s=1}^{I} p_s r_{sj}} .$$

The substitute for Eq. 4.17 in the Difference method is P_{ij}:

$$4.18)\quad P_{ij} = \frac{Rp_i r_{ij}(1-U_{ij})}{\sum_{s=1}^{I} Rp_s r_{sj}(1-U_{sj})} = \frac{p_i r_{ij}(1-U_{ij})}{\sum_{s=1}^{I} p_s r_{sj}(1-U_{sj})}$$

Inspection of Eq. 4.18 shows that only when the exploitation rates among ages within a run will P_{ij} be an unbiased estimate of the frequency of age i in run j and the Difference method provide accurate allocations.

Because the Difference method has several versions depending on the number of age groups and the number of runs in a situation, a detailed investigation of the bias in the Difference method, like the one for the Pooled method, would not be as simple. In lieu of a detailed investigation of its bias, examples of allocation schemes made with the Difference method are used to show bias.

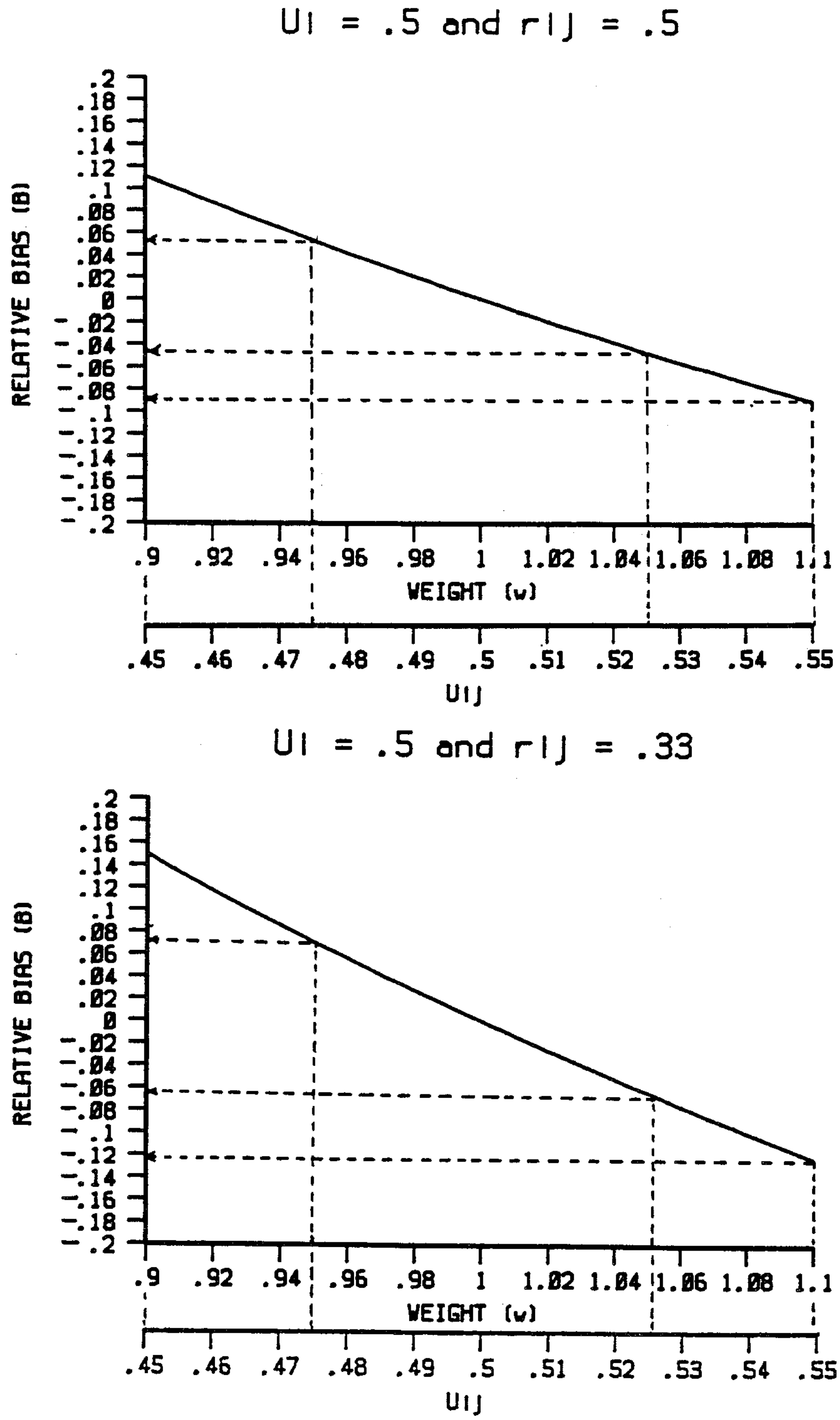

Figure 4.2 Relative bias in estimated catch allocation made with the Pooled method under typical exploitation rates and run compositions. The top graph shows bias for one of two equally sized runs with an exploitation rate of 50%. The bottom graph shows bias of one of two runs with the other runs twice as large and an overall exploitation rate of 50%.

5. EXAMPLES

Bristle Bay

The hypothetical River of No Return and the hypothetical River Styx each have a run of sockeye salmon which are fished by the Bristle Bay gill net fleet. Each run has two age groups, 4_2 and 5_2. The older fish from each river enter the fishery first with fish from the River of No Return entering before those headed for the River Styx. Members of each age group from each river take one month to pass any point in their journey to spawn, but there is a different lag time between fishing grounds and counting towers for each age group and river. All age groups have normally distributed migratory timing. The fishery is six weeks long starting 8 June with four days a week fished. The fleet fishes differentially on the run by age and river and daily instantaneous fishing mortality rates vary accordingly. With these characteristics and known run size by age by river, a pattern of escapement and catch was generated (Figure 5.1-2 and Appendix D).

Hypothetical stratified random sampling programs were developed to sample the age compositions and count escapements[1]. One catch sample of 600 fish is taken each fishing period (each week) to get estimates of age composition of the catch. Six samples of 500 fish each and five samples of 600 fish each are taken from the rivers of No Return and Styx, respectively. Escapements to each river are estimates from expanded tower counts of ten minutes in length once every hour 24 hours a day for the entire season. Because the values in the hypothetical example are generated, they lack the random variation observed in actual situations, and their variances (Table 5.1) are smaller than can be expected for actual situations.

The comparative statistics between the allocation scheme for Bristle Bay made with the Pooled method and the true split among the catch shows that the allocations are biased (Table 5.2 and 5.3). Relative bias ranged from a 6% deviation for two-ocean salmon from the River Styx to 13% for three-ocean salmon from the River of No Return. Exploitation rates are highest for younger fish from the River of No Return and lowest for older fish of the same River.

Bias in allocation caused by poor sampling for age composition and by dissimilar exploitation rates are evident in Table 5.3. The hypothetical sampling program of the escapement to the River Styx is inadequate to precisely estimate the rapidly changing age composition, and subsequently the strength of two-ocean fish to that river is underestimated by 6%. And because two-ocean fish far outnumber three-ocean fish in this river, the small bias for the former age group becomes a 54% bias in the estimate of escapement for the latter group. In the River of No Return, the hypothetical sampling program is sufficient to keep the estimates of age composition within 5% of their true values.

The sampling-induced bias occludes the bias in the allocations caused by dissimilar exploitation rates. When one run is more heavily exploited than another, salmon in the catch are incorrectly allocated to the other run. Bias for the more

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[1] These sampling programs are provided solely to demonstrate the procedures for catch allocation and are not an adequate sampling design for this hypothetical resource.

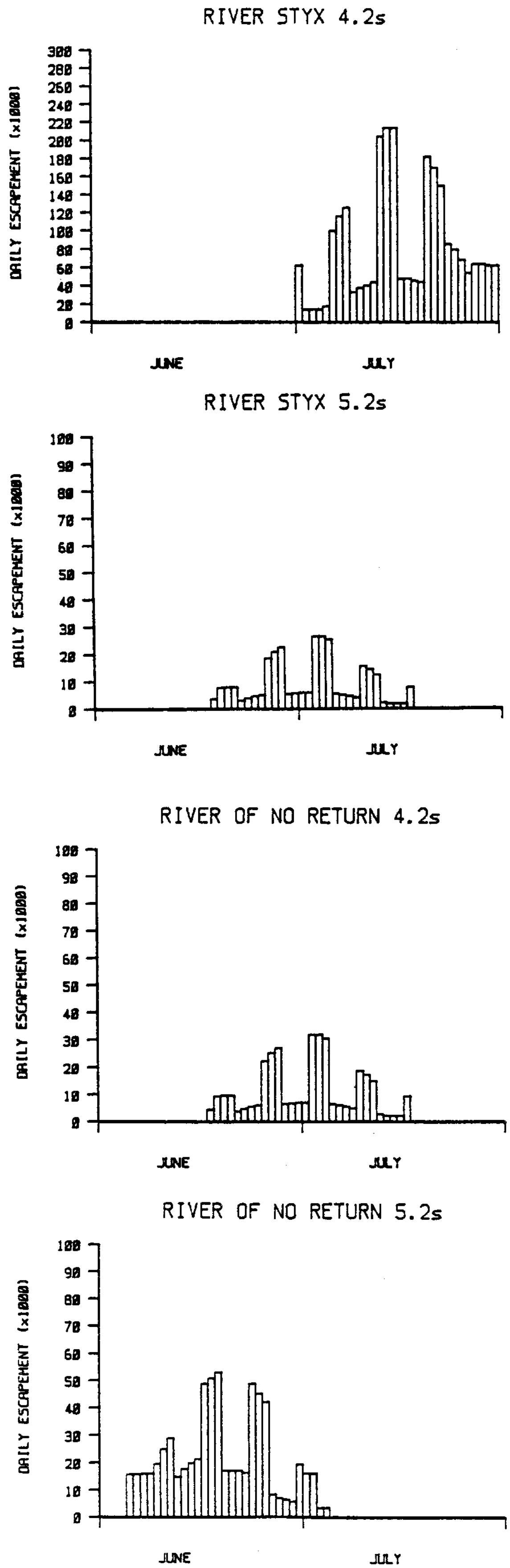

Figure 5.1 Hypothetical escapement by age by river to rivers in Bristle Bay (exact statistics).

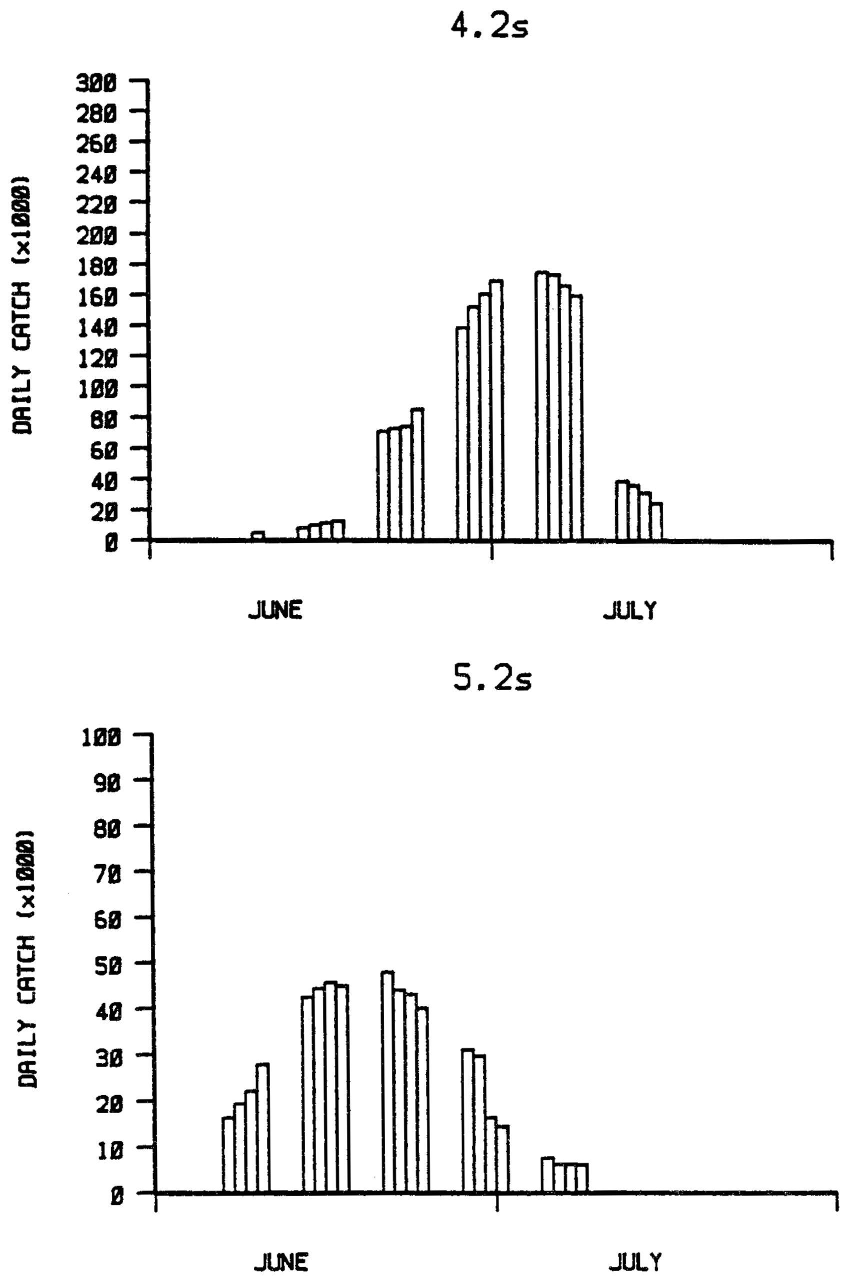

Figure 5.2 Hypothetical catch by age from Bristle Bay (exact statistics).

Table 5.1. Escapements, catches, and sampling information for the hypothetical fishery in Bristle Bay. Fishing periods correspond to the sampling strata. Variances of escapements are from simulated partial counts from towers; CVs for partial counts approximate 2% as listed in Becker (1962). Percents correspond to p_{ijh}, numbers caught or escaped to $c_{..k}$ or $E_{.jh}$, variances to $V[E_{.jh}]$, sampling periods to h or k, and number of fish sampled to N_{jh} or N_k.

Period Number	Fishing Periods Sampling Periods	Number of Fish Sampled	Count 4_2	Count 5_2	Percent 4_2	Percent 5_2	Number Caught or Escaped	Variance of Number Caught or Escaped
River Styx Escapements								
1	6/1-27	600	0	600	0	100	63,236	567,015
2	6/28-7/6	600	0	600	0	100	268,565	38,588,372
3	7/7-15	600	533	67	89	11	979,733	407,194,400
4	7/16-24	600	600	0	100	0	995,031	427,892,768
5	7/25-31	600	600	0	100	0	455,938	7,259,968
							2,762,503	
Escapements to River of No Return								
1	6/1-12	500	0	500	0	100	137,000	8,150,578
2	6/13-19	500	0	500	0	100	241,000	49,610,292
3	6/20-26	500	114	386	23	77	267,183	39,221,176
4	6/27-7/3	500	275	225	55	45	191,216	19,041,456
5	7/4-10	500	500	0	100	0	111,661	22,431,722
6	7/11-17	500	500	0	100	0	50,830	5,315,020
							998,890	
Catch								
1	6/8-11	600	90	510	15	85	91,582	
2	6/15-18	600	132	468	22	78	220,359	
3	6/22-25	600	409	191	68	32	480,929	
4	6/29-7/2	600	543	57	91	9	713,788	
5	7/6-9	600	573	27	96	4	699,516	
6	7/13-16	600	600	0	100	0	131,213	
							2,337,387	

Table 5.2. Allocation scheme for the hypothetical fishery in Bristle Bay made with the Pooled method. The variances for estimated escapements are ignored to make allocations based on known escapements.

Escapements to River Styx:

	$\hat{E}_{ij}$	Known $V[\hat{E}_{ij}]$	Estimated $V[\hat{E}_{ij}]$
4_2	2,322,931	156,785,472	914,543,360
5_2	439,572	156,785,472	200,934,432

Escapements to the River of No Return:

	$\hat{E}_{ij}$	Known $V[\hat{E}_{ij}]$	Estimated $V[\hat{E}_{ij}]$
4_2	329,112	43,376,448	78,981,352
5_2	669,778	43,376,448	128,270,768

Catches:

	$\hat{C}_i$	$V[\hat{C}_i]$
4_2	1,841,543	200,509,712
5_2	495,844	200,509,712

	River Styx Allocation|Return $\hat{C}_{ij}\|\hat{R}_{ij}$	River of No Return Allocation|Return $\hat{C}_{ij}\|\hat{R}_{ij}$
4_2	1,613,012|3,935,943	228,531|557,643
5_2	196,475|636,047	299,369|969,147
Standard Errors (Escapement Known):		
4_2	13,078|17,965	4,505|10,682
5_2	6,654|16,821	9,267|11,035
Standard Errors (Escapement Estimated):		
4_2	13,778|34,692	6,253|14,162
5_2	7,082|18,748	9,579|15,368

Table 5.3. Effect of imprecision in the estimates of escapement age composition in the hypothetical fishery in Bristle Bay on allocations made with the Pooled method.

Age	River Styx	River of No Return
Estimated vs. actual catch by age by river:		
	$\hat{C}_{ij} \mid C_{ij}$	$\hat{C}_{ij} \mid C_{ij}$
4_2	1,613,012 | 1,523,053	228,531 | 256,251
5_2	196,475 | 213,544	299,369 | 344,539
Relative bias in estimated catch by age by river vs. exploitation rate by age by river:		
	$B_{ij} \mid U_{ij}$	$B_{ij} \mid U_{ij}$
4_2	.06 | .38	-.11 | .47
5_2	-.08 | .43	-.13 | .34
Estimated vs. actual escapement by age by river:		
	$\hat{E}_{ij} \mid E_{ij}$	$\hat{E}_{ij} \mid E_{ij}$
4_2	2,322,931 | 2,476,147	329,112 | 343,629
5_2	439,572 | 286,356	669,778 | 655,261
Relative bias in estimates of escapement by age by river:		
	$(\hat{E}_{ij}-E_{ij})/E_{ij}$	$(\hat{E}_{ij}-E_{ij})/E_{ij}$
4_2	-.06	-.04
5_2	.54	.02
Relative bias in estimated catch by age by river with estimated and with actual escapements by age by river:		
	B_{ij} Estimated | Actual	B_{ij} Estimated | Actual
4_2	+.06 | +.06	-.11 | -.12
5_2	-.08 | -.21	-.13 | +.13

exploited run is negative, while bias for the less exploited run is positive. When the known escapements and catches are used in the allocation instead of their estimates, the bias caused by sampling is removed from the allocation, and only the bias caused by dissimilar exploitation rates remains (Table 5.3). When the sampling-induced bias in escapements is removed, the biases are exactly as predicted by Eq. 4.14. The sampling-induced bias in this example has little effect on two-ocean fish; the three-ocean fish are the more affected group.

The catch allocation scheme made with the Difference method is also biased (Table 5.4 and 5.5). The two-ocean age class from the River Styx is allocated too many fish while all other age groups are allocated too few. When the exploitation rate on a major component of the run is low relative to the rates on the other components, the Difference method pulls fish away from the other components. If exact statistics on escapements are used, the biases decline but have the same general pattern. And if exact statistics on both catches and escapements are used, the biases are reduced further but are still present.

The average bias in the results from the Difference method declines as the precision of its inputs improves while no such advantage occurs for results from the Pooled method (Tables 5.6-7). The absolute values of the relative biases when averaged within each allocation scheme drop from 28% to 12% for the Difference method but vary only from 12% to 10% to 13% for the Pooled method. Increases in precision and accuracy of sampling programs improve the efficacy of the Difference method, but only to the level attained with the Pooled method.

Lynn Canal

The drift gill net fishery in Lynn Canal, an arm of the Pacific Ocean in Southeastern Alaska, captures sockeye salmon bound for the Chilkat and the Chilkoot Rivers (Figure 5.3). In 1982, most early catches in the fishery were fish aged 5_2 with other age groups, mostly 4_2, 6_2, and 5_3, making up later catches (Table 5.8 and Figure 5.4). Management of the fishery was designed to ensure escapements and was conducted through time and area closures for a fleet that grew as the season progressed (Figure 5.5). Sampling programs at local canneries were used to estimate the age composition of the catch. Sockeye salmon surviving the fishery passed through weirs on both rivers and were all counted. Sampling programs at each weir were used to estimate the age composition of escapements. The run of sockeye salmon to Chilkat Lake was composed equally of fish aged 5_2, 5_3, and 6_3, while the run to Chilkoot Lake was composed of two parts fish aged 5_2 and one part 4_2 (Figure 5.6). Both runs reached the fishery at about the same time. Travel times between the fishery and the lakes was about 5 days for the Chilkoot run and about 30 days for the Chilkat. Fish passed through the weir on the Chilkoot River with biomdal frequency and through the weir on the outlet of Chilkat Lake sporadically (Figure 5.6). Although the migrational periodicity at Chilkoot weir was probably due to the migrational timing of two stocks in the run, the periodicity at Chilkat weir was due to physical factors. After frequent, intense rainfalls the Tsirku River overflowed into the outlet from Chilkat Lake causing the flow to reverse through the weir for a few days at a time. While the flow was reversed, no fish passed through the weir.

Scale pattern analysis with linear discriminant functions has been used for several years to separate catches of sockeye salmon in Lynn Canal according to their

Table 5.4. Allocation scheme for the hypothetical fishery in Bristle Bay made with the Difference method.

Escapements to River Styx:

	$\hat{P}_{ij}$	$V[\hat{P}_{ij}]$
4_2	.841	$.2055 \times 10^{-4}$
5_2	.159	$.2055 \times 10^{-4}$

Escapements to the River of No Return:

	$\hat{P}_{ij}$	$V[\hat{P}_{ij}]$
4_2	.329	$.4347 \times 10^{-4}$
5_2	.671	$.4347 \times 10^{-4}$

Catches:

	$\hat{Q}_i$	$V[\hat{Q}_i]$
4_2	.788	$.3670 \times 10^{-4}$
5_2	.212	$.3670 \times 10^{-4}$

	River Styx Allocation | Return $\hat{C}_{ij} \| \hat{R}_{ij}$	River of No Return Allocation | Return $\hat{C}_{ij} \| \hat{R}_{ij}$
4_2	1,762,257 | 4,085,188	79,604 | 408,716
5_2	333,174 | 772,746	162,353 | 823,131
Standard Errors (Escapement Known):		
4_2	26,696 | 37,808	5,635 | 7,850
5_2	5,317 | 10,547	22,444 | 23,972
Standard Errors (Escapement Estimated):		
4_2	26,696 | 59,904	5,635 | 10,238
5_2	5,317 | 13,726	22,444 | 27,466

Table 5.5. Effect of imprecision in the estimates of escapement age composition in the hypothetical fishery in Bristle Bay on allocations made with the Difference method.

Age	River Styx	River of No Return
Estimated vs. actual catch by age by river:		
	$\hat{C}_{ij} \| C_{ij}$	$\hat{C}_{ij} \| C_{ij}$
4_2	1,762,257 | 1,523,053	79,604 | 256,251
5_2	162,353 | 213,544	333,174 | 344,539
Relative bias in estimated catch by age by river vs. exploitation rate by age by river:		
	$B_{ij} \| U_{ij}$	$B_{ij} \| U_{ij}$
4_2	.16 | .38	-.69 | .47
5_2	-.24 | .43	-.03 | .34
Estimated vs. actual escapement by age by river:		
	$\hat{E}_{ij} \| E_{ij}$	$\hat{E}_{ij} \| E_{ij}$
4_2	2,322,931 | 2,476,147	329,112 | 343,629
5_2	439,572 | 286,356	669,778 | 655,261
Relative bias in estimates of escapement by age by river:		
	$(\hat{E}_{ij}-E_{ij})/E_{ij}$	$(\hat{E}_{ij}-E_{ij})/E_{ij}$
4_2	-.06	-.04
5_2	.54	.02
Relative bias in estimated catch by age by river with estimated and with actual escapements by age by river:		
	B_{ij} Estimated | Actual	B_{ij} Estimated | Actual
4_2	+.16 | +.11	-.69 | -.39
5_2	-.24 | -.07	-.03 | -.13

Table 5.6. Relative biases in allocations from the Pooled and the Difference methods when exact statistics on catches and escapements to Bristle Bay are used.

Actual escapement by age by river and actual proportion by age within each escapement:

	River Styx	River of No Return
	$E_{ij} \| P_{ij}$	$E_{ij} \| P_{ij}$
4_2	2,476,147|.895	343,629|.344
5_2	286,356|.105	655,261|.656

Actual catch by age and actual proportion by age of the catch:

4_2	1,779,304|.761
5_2	558,083|.239

Actual and estimated catch allocations by age by river with actual catches, escapements, and their proportions:

	Pooled|Difference|Actual	Pooled|Difference|Actual
4_2	1,562,471|1,583,208|1,523,053	216,833|195,543|256,251
5_2	169,719| 185,740 |213,544	388,364|372,896|344,539

Relative bias in estimated catch by age by river with actual catches, escapements, and their proportions:

	B_{ij}	B_{ij}
	Pooled|Difference	Pooled|Difference
4_2	+.03|+.04	-.15|-.24
5_2	-.21|-.13	+.13|-.08

Table 5.7. Average absolute values of relative biases in allocations from the Pooled and the Difference methods with different levels of accuracy on catch and escapement statistics from Bristle Bay.

	Pooled	Difference
Actual statistics	.13	.12
Actual escapement statistics/ estimated catch statistics	.10	.18
Estimated statistics	.12	.28

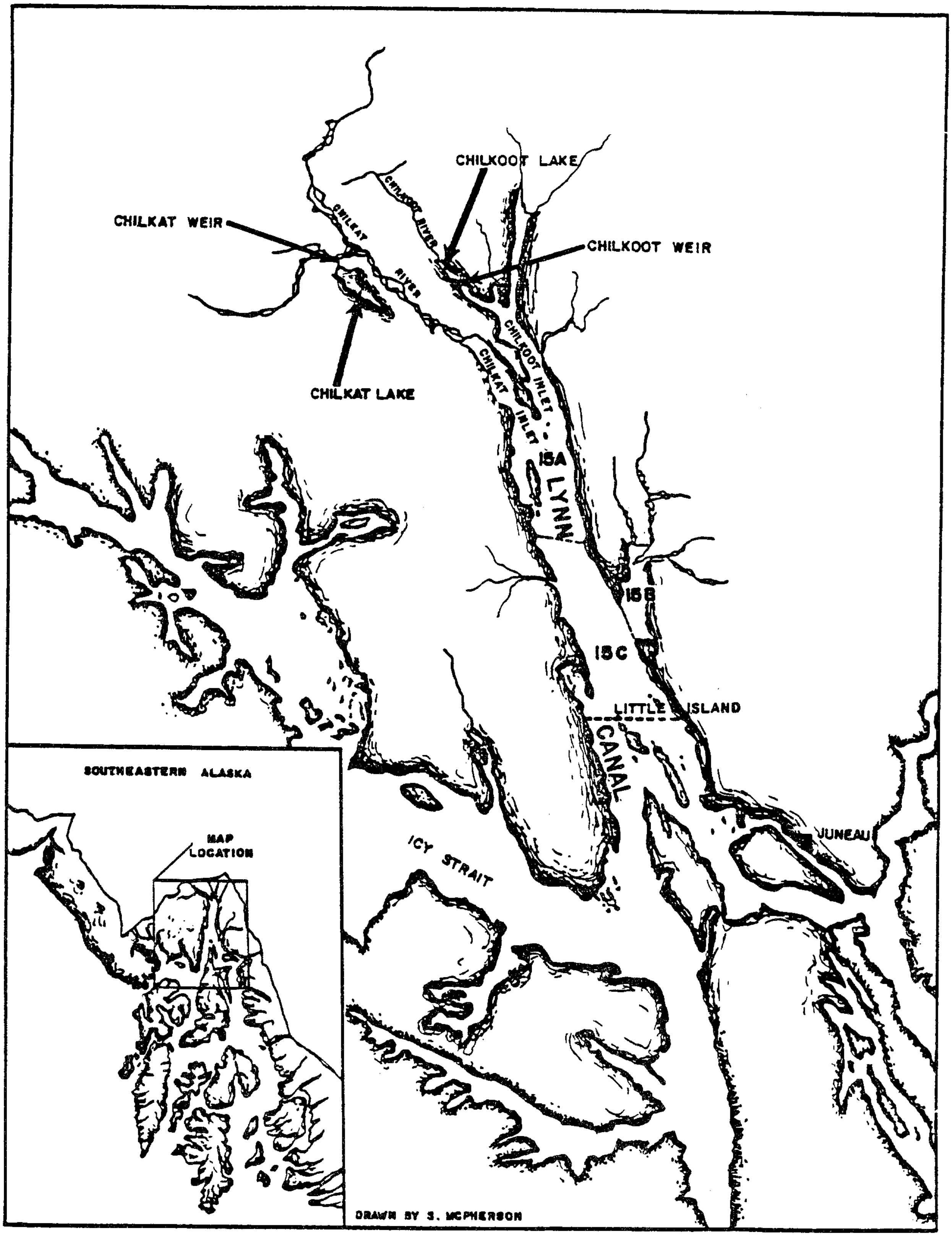

Figure 5.3 Map of Lynn Canal with inset of Southeastern Alaska.

Table 5.8. Escapements, catches, and sampling information for the drift gill net fishery for sockeye salmon in Lynn Canal. Fishing and sampling periods are considered strata. Data are from McPherson et al. (1983).

Period Number	Fishing Periods Sampling Periods	Number of Fish Sampled	Count 5_2	Count Others	Percent 5_2	Percent Others	Number Caught or Escaped
Escapements to Chilkoot River							
1	6/11-7/18	560	518	42	92.5	7.5	33,063
2	7/19-30	556	421	135	75.7	24.3	35,505
3	7/31-9/8	562	385	177	68.5	31.5	34,405
							102,973
Escapements to Chilkat River							
1	6/28-9/4	582	190	392	32.6	67.4	20,691
2	9/5-16	516	9	507	1.7	98.3	24,404
3	9/17-10/3	517	9	508	1.7	98.3	35,126
							80,221
Catch							
1	6/13-14	320	302	26	94.4	5.6	2,370
2	6/20-23	321	277	44	86.2	13.8	6,181
3	6/27-30	333	286	47	85.9	14.1	8,023
4	7/4-7	328	281	47	85.7	14.3	8,399
5	7/11-13	334	290	44	86.8	13.2	8,386
6	7/18-20	369	320	49	86.8	13.2	12,622
7	7/25-28	350	259	91	74.0	26.0	47,954
8	8/1-3	434	247	187	56.9	43.1	55,736
9	8/8-13	471	207	264	44.0	56.0	52,184
10	8/16-20	441	217	224	49.2	50.8	30,667
11	8/22-27	527	168	359	31.9	68.1	24,512
12	8/29-31	353	34	319	9.6	90.4	11,429
13	9/5-7	285	110	175	38.6	61.4	2,734
14	9/12-10/17	471	23	448	4.9	95.1	2,331
							273,528

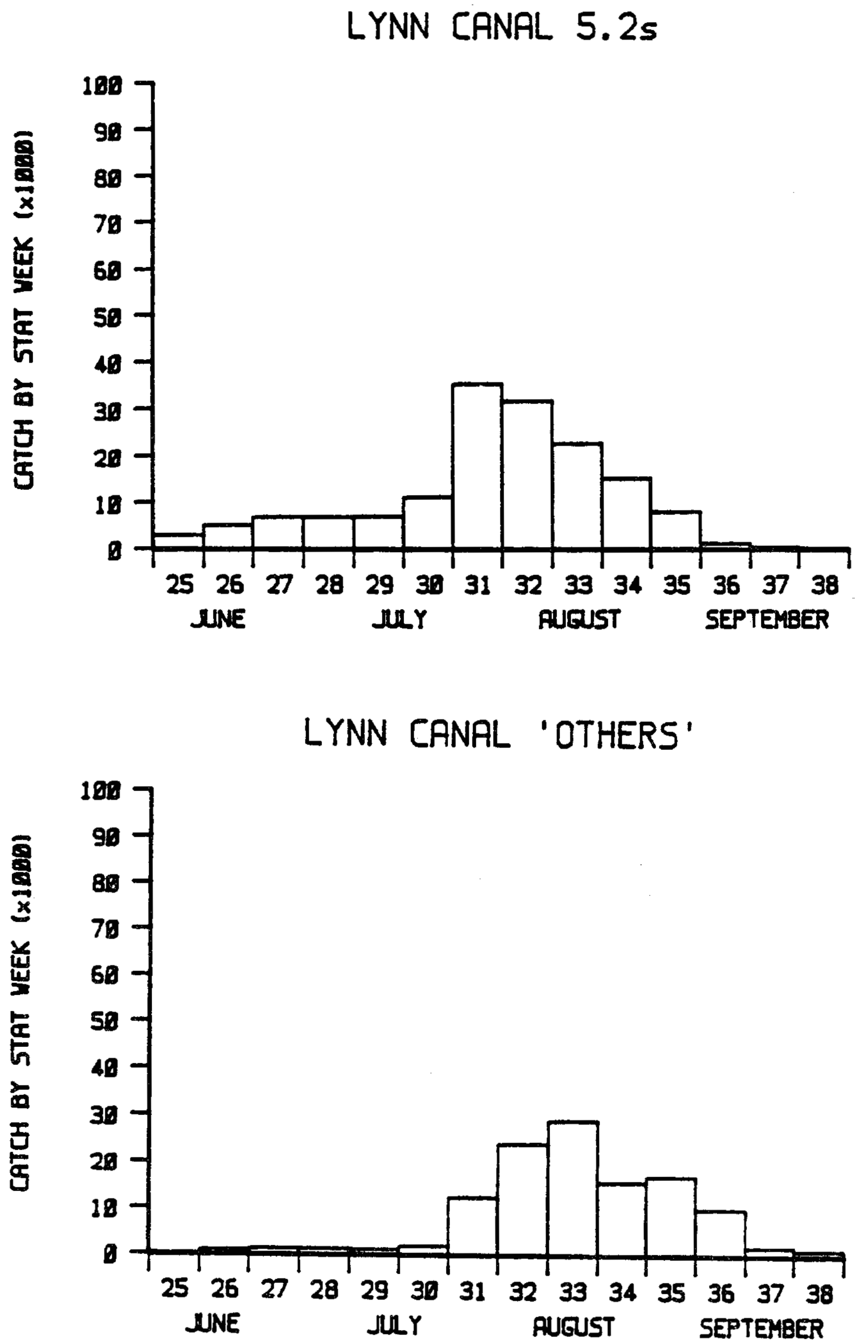

Figure 5.4 Catch by age of sockeye salmon in the drift gill net fishery in Lynn Canal during 1982. Numbers along abscissa are statistical weeks.

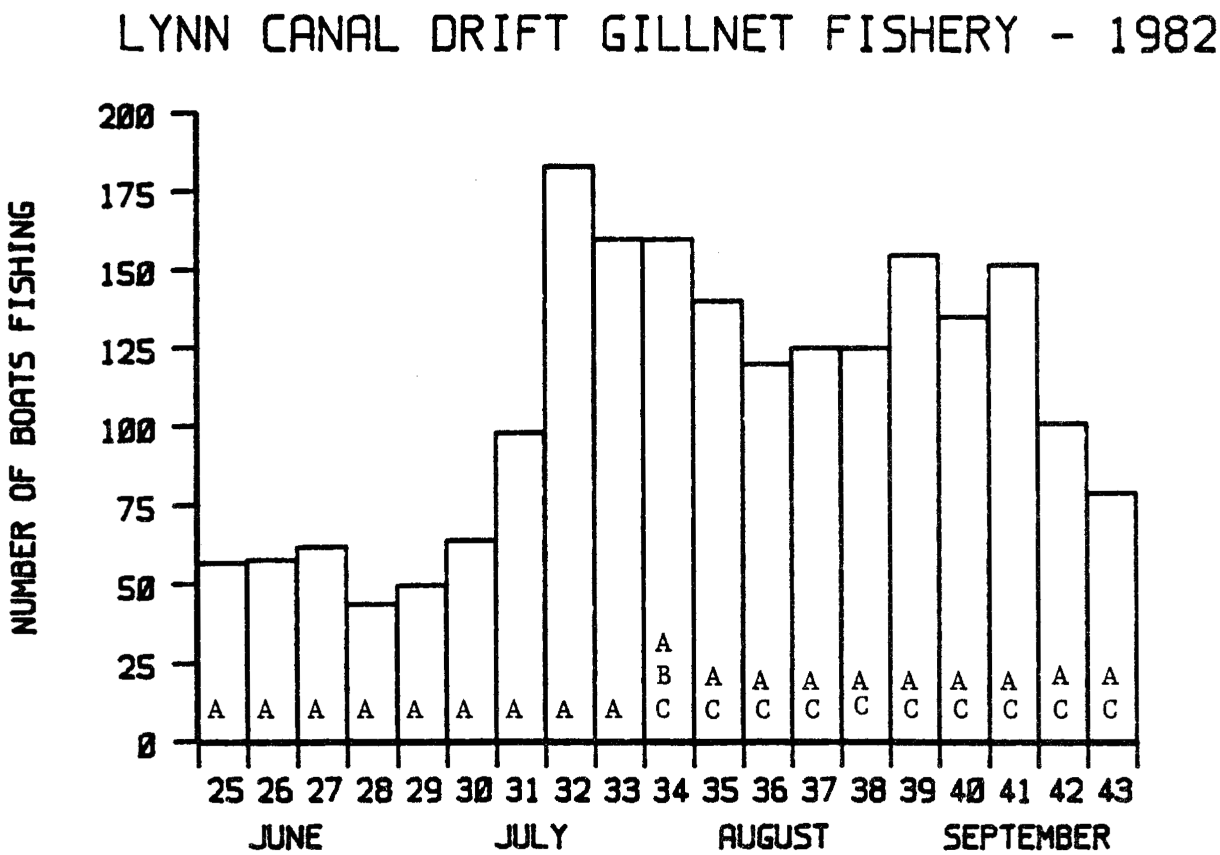

Figure 5.5 Fishing effort by statistical week in 1982 in the drift gill net fishery for sockeye salmon in Lynn Canal during 1982. Numbers along the abscissa are statistical weeks. Letters in the columns are the subdistricts in District 15 open that week.

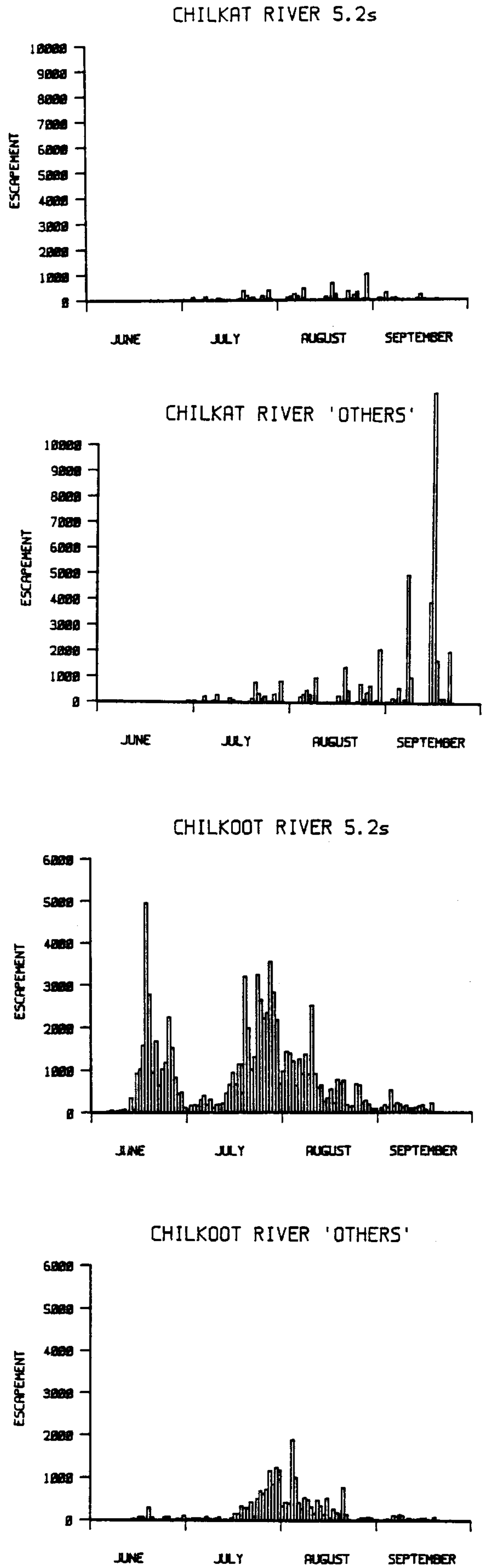

Figure 5.6 Escapements by age of sockeye salmon into the Chilkat and the Chilkoot Rivers in 1982.

rivers of origin. In 1982, McPherson et al. (1983) divided the catch according to age and allocated the catch of fish aged 5_2 with linear discriminant analysis. The fishery that year harvested 131,861 sockeye salmon bound for the Chilkoot River and 23,092 bound for the Chilkat River. Although variances for these figures are not given, 90% confidence intervals for the run fractions of Chilkoot fish for each statistical week were calculated according to methods described in Pella and Robertson (1979). Although they are not variances of the allocation, the weighted average of confidence intervals in McPherson et al. (1983, Table 11, p. 25) approximates a 90% confidence interval for the allocation. The weighted average of the confidence intervals for 1982 is ±6% for both allocations.

For this example, results of scale pattern analysis for 1982 are assumed to be the "known" when compared against allocations made with the Pooled and with the Difference methods. Because McPherson et al. (1983) directly allocated only fish aged 5_2, the catch and escapements in this example are divided into age groups 5_2 and Others. Table 5.8 contains catch and escapement information used in the Pooled and in the Difference methods of catch allocation.

When compared with allocations made with scale pattern analysis, the allocations made with the Pooled method are 7% too high for the Chilkoot River and 42% too low for the Chilkat (Table 5.9-10). Fish headed for the Chilkat River are more exploited than those headed to the Chilkoot, and as predicted in Section 4 and shown in the hypothetical Bristle Bay example, the less exploited run is allocated too many fish at the expense of the more exploited run. The allocation of fish aged 5_2 to the Chilkoot River is 140,818 fish with a 2,242 fish standard error. Because the escapement is completely counted, $V[E_{.j}] = 0$ which probably accounts in part for the excellent precision of the allocation -- the coefficient of variation (CV) is 1.6%. The allocation to the Chilkat River is 13,481 with a standard error of 779 (CV = 5.8%). Assuming that both allocations are normally distributed, the 90% confidence intervals around the allocations to the Chilkoot River and to the Chilkat River are ±3% and ±10% of the allocations, respectively.

Allocations made with the Difference method, like those from the Pooled method, are too high for the Chilkoot River (9%) and too low for the Chilkat River (61%) (Table 5.11-12). The Chilkoot River is allocated 144,058 with a standard error of 3,426 (CV = 2.4%), and the Chilkat River is allocated 9,117 fish with a standard error of 223 fish (CV = 2.4%). Again, the standard errors of the allocations are small because of the complete escapement counts on both rivers. Assuming both allocations are normally distributed, 90% confidence intervals around the allocations to the Chilkoot River and to the Chilkat River are both ±4% of their allocations.

The allocations made with scale pattern analysis, the Pooled method, and the Difference method are not significantly different for the Chilkoot River but are grossly different for the Chilkat (Table 5.13). While the confidence intervals of the low and high allocations to the Chilkoot River overlap, none of the confidence intervals for allocations to the Chilkat River do.

Although the Pooled and the Difference methods produced allocation schemes with excellent precision, these schemes had poor accuracy. Both methods allocated too many fish to the less exploited (u = .62) and larger (85%) Chilkat run at the expense of the smaller (15%) and more exploited (u = .75) Chilkat run. Why

Table 5.9. Allocation scheme for the 1982 Lynn Canal fishery for sockeye salmon made with the Pooled method. Information on catches, escapements, and their age compositions are from McPherson et al. (1983).

Escapements at the weir on the Chilkoot River:

	$\hat{E}_{ij}$	$V[\hat{E}_{ij}]$
5_2	81,028	992,520
Others	21,945	992,520

Escapements at the weir on the Chilkat River:

	$\hat{E}_{ij}$	$V[\hat{E}_{ij}]$
5_2	7,757	215,649
Others	72,464	215,649

Catches in the Lynn Canal gillnet fishery:

	$\hat{C}_i$	$V[\hat{C}_i]$
5_2	154,299	5,353,553
Others	118,509	5,353,553

	Chilkoot River Allocation|Return $\hat{C}_{ij}\|\hat{R}_{ij}$	Chilkat River Allocation|Return $\hat{C}_{ij}\|\hat{R}_{ij}$
5_2	140,818|221,846	13,481|21,238
Others	27,547|49,492	90,962|163,426
Standard Errors:		
5_2	2,242|2,514	779|1,228
Others	1,109|2,034	2,034|2,106

Table 5.10. Effect of imprecision in the estimates of escapement age composition in the drift gill net fishery for sockeye salmon in Lynn Canal during 1982 on allocations made with the Pooled method. Allocations were made with the Pooled method and were compared with allocations made with scale pattern analysis as reported in McPherson et al. (1983). Exploitation rates are calculated from allocations made from McPherson. Because McPherson allocated only the catch of sockeye salmon aged 5_2, no other age groups are represented in the table.

Age	Chilkoot River	Chilkat River
Pooled method vs. scale pattern analysis:		
	$\hat{C}_{ij} \vert C_{ij}$	$\hat{C}_{ij} \vert C_{ij}$
5_2	140,818 | 131,861	13,481 | 23,092
Relative bias in estimated catch by age by river vs. exploitation rate by age by river:		
	$B_{ij} \vert U_{ij}$	$B_{ij} \vert U_{ij}$
5_2	+.07 | .62	-.42 | .75

Table 5.11. Allocation scheme for the 1982 Lynn Canal fishery for sockeye salmon made with the Difference method. Information on catches, escapements, and their age compositions are from McPherson et al. (1983).

Escapements at the weir on the Chilkoot River:

	$\hat{P}_{ij}$	$V[\hat{P}_{ij}]$
5_2	.79	$.9360 \times 10^{-4}$
Others	.21	$.9360 \times 10^{-4}$

Escapements at the weir on the Chilkat River:

	$\hat{P}_{ij}$	$V[\hat{P}_{ij}]$
5_2	.10	$.3351 \times 10^{-4}$
Others	.90	$.3351 \times 10^{-4}$

Catches in the Lynn Canal gillnet fishery:

	$\hat{Q}_i$	$V[\hat{Q}_i]$
5_2	.566	$.7156 \times 10^{-4}$
Others	.434	$.7156 \times 10^{-4}$

	Chilkoot River Allocation | Return $\hat{C}_{ij} \mid \hat{R}_{ij}$	Chilkat River Allocation | Return $\hat{C}_{ij} \mid \hat{R}_{ij}$
5_2	144,058 | 225,086	9,117 | 16,874
Others	38,294 | 60,239	82,059 | 154,523
Standard Errors:		
5_2	3,426 | 4,093	223 | 674
Others	622 | 1,109	2,182 | 2,244

Table 5.12. Effect of imprecision in the estimates of escapement age composition in the drift gill net fishery for sockeye salmon in Lynn Canal during 1982 on allocations made with the Difference method. Allocations were made with the Difference method and were compared against allocations made with scale pattern analysis as reported in McPherson et al. (1983). Exploitation rates are calculated from allocations made from McPherson. Because McPherson allocated only the catch of sockeye salmon aged 5_2, no other age groups are represented in the table.

Age	Chilkoot River	Chilkat River
Difference method vs. scale pattern analysis:		
	$\hat{c}_{ij} \mid c_{ij}$	$\hat{c}_{ij} \mid c_{ij}$
5_2	144,058 | 131,861	9,117 | 23,092
Relative bias in estimated catch by age by river vs. exploitation rate by age by river:		
	$B_{ij} \mid U_{ij}$	$B_{ij} \mid U_{ij}$
5_2	+.09 | .62	-.61 | .75

Table 5.13. Relative biases in allocations from the Pooled and the Difference methods when compared with exact statistics from the return to Lynn Canal in 1982 as generated with scale pattern analysis from McPherson et al. (1983). Because McPherson allocated only the catch of sockeye salmon aged 5_2, no other age groups are represented in the table.

Actual and estimated catch allocations by age by river with actual catches, escapements, and their proportions:

	Chilkoot River			Chilkat River		
	Pooled	Difference	Scales	Pooled	Difference	Scales
5_2	140,818	144,058	131,861	13,481	9,117	23,092

Relative bias in estimated catch by age by river with actual catches, escapements, and their proportions:

	B_{ij}		B_{ij}	
	Pooled	Difference	Pooled	Difference
5_2	+.07	+.09	-.42	-.61

Overlap in 90% confidence intervals of the allocations:

```
Scale pattern analysis  *------                                   } Chilkoot
Pooled method               ---*---                               }  River
Difference method             ----*----                           }

{ *------   Scale pattern analysis                                } Chilkat
{           Pooled method        ----------*----------            }  River
{           Difference method                        ----*----    }
```

---

the Chilkat run of 5_2 age fish was more exploited in 1982 than the Chilkoot run is not clear from the data presented in McPerson et al. (1983). Even though the runs take several months to pass through the fishery and the fishery is opened regularly 3-5 days a week, the area closures and the growth in fleet effort (Figure 5.5) during the season along with slight differences in migratory timing are probably the cause of the substantial difference in exploitation rates. Because the run of 5_2 aged fish to the Chilkoot River is much bigger than the other run, the allocation to it has small bias while the allocation to the Chilkat River has large bias.

6. DISCUSSION

Of the two methods of allocating catches based on the age composition of escapements, which is better, the Difference or the Pooled method? The Pooled method appears the better of the two because it is less sensitive to commonly occurring circumstances: gear selectivity, differences in age compositions, and sampling programs that provide imprecise estimates.

The Pooled method will always provide an allocation scheme while under some circumstances the Difference method will not. If there are more runs than age groups in a fishery, the Difference method will not work. Also, some difference in the age compositions must exist before the Difference method can be used. And these differences must be significant; any difference between age compositions must be greater than the precision of their estimates if allocation schemes from the Difference method are to be meaningful.

When differences between age compositions are small, the precision of the allocations from the Difference method is poor. Note that the equation for the variance of the allocation (Eq. 3.21) contains the reciprocal of the difference between fractions squared. As the difference gets smaller, the variance will increase quadratically. The Pooled method has no similar behavior.

The accuracy of allocation schemes from the Pooled method is less sensitive to the quality of sampling programs for age composition than is the accuracy from the Difference method. Errors in estimating the age compositions of escapements are doubly troublesome for the Difference method. Estimated age proportions are used twice in the Difference method, once to estimate the proportion of the catch destined for a specific river and again to partition those fish into age groups. The accuracy of the Difference method can be improved with better catch and escapement sampling programs, although the hypothetical example indicates the improvement is limited to about the level of accuracy obtained with the Pooled method. Unlike the other method, the Pooled method uses numbers, not proportions, which lessens the impact of inaccurate estimates by weighting each proportion by the magnitude of its escapement and by "averaging" inaccurate information from inadequate escapement and catch sampling programs with information from adequate programs to determine the denominator in the method $(\hat{E}_{i.})$.

The Pooled method is less sensitive to the effect of gear selectivity on the age composition of the escapement than is the Difference method. Allocations from the Pooled method for each age group are calculated independently of allocations for the other age groups while allocations from the Difference method are not. Only when there is little or no gear selectivity or when fish from different runs of the same age are significantly different in size will the Pooled method lose this advantage over the Difference method.

And finally, the Pooled method is easier to understand. The premise behind the Pooled method is simple: if X% of the escapement of fish of age Y are in run Z, X% of the fish of age Y in the catch are members of run Z. The bias in the Pooled method is also obvious: if all the fish of age Y headed for run Z are caught, X=0 and no fish of age Y are allocated to run Z. This obvious bias in the Pooled method and the apparent lack of bias in the Difference method has largely resulted in a preference for the latter method.

Although the Pooled method is better than the Difference method, it is still not an adequate method when exploitation rates are not the same for all runs, which unfortunately occurs quite often. If all runs have the same timing and are completely mixed over the fishing grounds, the exploitation rates must be the same for all runs and the Pooled method provides unbiased allocations (provided the escapement and catch sampling programs are adequate). For the Difference method to provide accurate allocations, all of these conditions must hold and in addition there must be no gear selectivity. If runs have different migratory timing, exploitation rates can still be the same for all runs if the fishing effort is constant throughout the season (and of course no gear selectivity occurs if the Difference method is to be used.) Unfortunately, migratory timing is usually different for different runs as is the timing of age groups within runs. And constant fishing effort permits no latitude in management of the fishery which is an unacceptable condition. In fact, any management policy to exploit one run greater than another will, if successful, create different exploitation rates for different runs and will make any allocation of catch with the Pooled or Difference methods biased.

If the Pooled method should not be used in some circumstances, in what circumstances can it be used with minimal bias? First, the bias in allocations can be corrected for a fishery in which the exploitation rates, fishing effort, and catchability information is known independently of age composition. The "Catch-22" in these circumstances is that if the exploitation rates are known, the allocation is a simple division of the number escaped and the complement of the exploitation rate; the Pooled method is not needed.

Second, bias in allocations could be negligible when the fishery is periodic with constant effort and fishes on a series of runs that take a long time to pass through the fishing grounds. For instance, if a fishery has 100 boats that fish on Mondays and Thursdays for 36 hours each period, each run is exposed to the same effort and suffers about the same exploitation rates. But if gear is changed during the fishing season, the effective effort will change and so will be exploitation rates if migratory timing is different among the runs. Also, if runs quickly pass through the fishing grounds, the days they pass through relative to fishing dates will greatly change their exploitation rates. For instance, if two runs pass through at different times, each taking five days, one run could pass between two fishing periods and not be fished while the other pass through during a period and be fished. If the runs are drawn out, this potential problem is minimal. The gill net fishery in Lynn Canal is a counterexample. The runs of sockeye salmon passed slowly through Lynn Canal, and the fishery opened regularly, yet exploitation rates were different between the runs and the Pooled method gave biased allocations. The effort in the

Lynn Canal fishery changed in magnitude and in area of application as the season progressed in 1982. If the Pooled method is to give allocations with small bias in these circumstances, management must be minimal (few area closures) and effort must be constant.

Third, bias in allocations will be small when the fishery is small. If only a small portion of the runs are taken, any effect the fishery will have in the age composition of the escapement will be minimal. As a rule of thumb, the bias will be negligible if the exploitation rate is 15% or below (see Figure 4.1).

And finally, the Pooled method will provide allocations with negligible relative bias for the greatly dominant run in the fishery. Even when exploitation rates are grossly different, the bias in large runs is relatively small, as is the case for the Lynn Canal example. However, the relative bias for the remaining small runs is huge. For instance, the Pooled method is not a bad choice to allocate fish aged 5_2 in the Lynn Canal fishery to the Chilkoot River run of sockeye salmon, but it is a terrible choice to allocate fish of the same age to the Chilkat River run. If in a mixed-run fishery one run is always much larger than others within an age group, the Pooled method can be used year after year with limited bias and no allocation made for the smaller runs. But if the dominance annually switches from run to run, the Pooled method will create brood tables for each run with alternating large and small biases.

If methods of allocating catches based on age composition of escapements have these problems, why bother with them at all? Because all other means of allocating catches also have problems. Scale pattern analysis will work only when there are differences in scale patterns. Tagging programs are expensive, work well only when tags are distributed or collected randomly with little tag loss and little induced mortality, and rely on some age composition information from escapements. And migratory timing studies require that escapement enumeration occur close enough to fishery to reflect openings and closings and require age composition information from escapements. There is no one best method for all fisheries. The best method is the simplest one that gives the best accuracy and best precision under the circumstances. There will be measurement error with all methods. The objective is to find the combination of methods that minimizes this error.

ACKNOWLEDGMENTS

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APPENDICES

APPENDIX A

The three FORTRAN subroutines in Appendix A calculate variances and allocations based on estimates of age compositions of catch and escapement with the Pooled method. Subroutine VARALL allocates catch by age to rivers, estimates run sizes by age by river, and calculates the variance of these allocations and run sizes. Subroutines AGCMP1 and AGCMP2 provide inputs (age compositions by age and their variances for either the catch or a single river) to VARALL. Both AGCMP subroutines are designed to use information from stratified random or stratified systematic sampling programs for age composition. AGCMP1 requires known population sizes for each stratum (i.e., the catch or an escapement completely counted), and AGCMP2 requires estimated population sizes and their variances for each stratum (i.e, expanded counts of an escapement). If an allocation is to be made from one catch to J rivers, the AGCMP subroutines are called J+1 times, once for the catch and once for each river; VARALL is called only once to make allocations and calculate run sizes. For each of the J+1 calls of AGCMP subroutines, call AGCMP1 when age compositions of the input populations are known or call AGCMP2 when age composition of the input population is estimated. Lists of outputs, inputs, and definitions of variables are included as comment statements for each subroutine. These subroutines are designed to run on Microsoft's FORTRAN-80 Language implemented on the VECTOR3 Microcomputer.

```
      SUBROUTINE VARALL(CI,CIJ,EIJ,IAGE,ISTK,RIJ,VCI,VCIJ,VEIJ,VRIJ)
C------------------------------------------------------------------------
C     "VARALL" ALLOCATES CATCHES TO RIVERS BY AGE AND CALCULATES RUN
C          SIZE BY AGE BY RIVER AND THEIR VARIANCES. INPUTS ARE CATCHES
C          BY AGE AND THEIR VARIANCES, ESCAPEMENTS BY AGE BY RIVER AND
C          AND THEIR VARIANCES, NUMBER OF AGES, AND NUMBER OF RIVERS.
C------------------------------------------------------------------------
      DIMENSION CI(10),CIJ(10,10),EI(10),EIJ(10,10),RIJ(10,10)
      DIMENSION VCI(10),VCIJ(10,10),VEI(10),VEIJ(10,10),VRIJ(10,10)
C------------------------------------------------------------------------
C     CI(I)         CATCH BY AGE I - INPUT
C     CIJ(I,J)      CATCH BY AGE I BY RIVER J - OUTPUT
C     EI(I)         ESCAPEMENT BY AGE I FOR ALL RIVERS
C     EIJ(I,J)      ESCAPEMENT BY AGE I BY RIVER J - INPUT
C     I,J           AGE,RIVER - SUBSCRIPTS
C     IAGE          NUMBER OF AGE GROUPS - INPUT
C     ISTK          NUMBER OF RIVERS - INPUT
C     RIJ(I,J)      RUN SIZE BY AGE I FOR RIVER J - OUTPUT
C     VCI(I)        VARIANCE OF CATCH BY AGE I - INPUT
C     VCIJ(I,J)     VARIANCE OF CATCH BY AGE I BY RIVER J - OUTPUT
C     VEI(I)        VARIANCE OF ESCAPEMENT BY AGE I FOR ALL RIVERS
C     VEIJ(I,J)     VARIANCE OF ESCAPEMENT BY AGE I BY RIVER J - INPUT
C     VRIJ(I,J)     VARIANCE OF RUN SIZE BY AGE I FOR RIVER J - OUTPUT
C------------------------------------------------------------------------
C     FIND ESCAPEMENT BY AGE AND ITS VARIANCE
C------------------------------------------------------------------------
```

```
      DO  20  J=1,ISTK
      T0=EIJ(I,J)/EI(I)
      T1=T0*T0
      T2=VEIJ(I,J)/EIJ(I,J)/EIJ(I,J)
      T3=VEI(I)/EI(I)/EI(I)
      T4=VEIJ(I,J)*2./EIJ(I,J)/EI(I)
      VEIJEI=(T2+T3-T4)*T1
C-----------------------------------------------------------------------
C     FIND THE ALLOCATION AND THE RUN SIZE
C-----------------------------------------------------------------------
      CIJ(I,J)=EIJ(I,J)*CI(I)/EI(I)
      RIJ(I,J)=EIJ(I,J)+CIJ(I,J)
C-----------------------------------------------------------------------
C     FIND THE VARIANCE OF THE ALLOCATION
C-----------------------------------------------------------------------
      T2=CI(I)*CI(I)*VEIJEI
      T3=VCI(I)*T1
      T4=VCI(I)*VEIJEI
      VCIJ(I,J)=T2+T3-T4
C-----------------------------------------------------------------------
C     FIND VARIANCE OF RUN SIZE
C-----------------------------------------------------------------------
      T1=EIJ(I,J)/EI(I)
      T2=TCIEI*T1
      T3=1.+TCIEI-T2
   20 VRIJ(I,J)=VCI(I)*T1*T1+VEIJ(I,J)*T3*T3+T2*T2*(VEI(I)-VEIJ(I,J))
      RETURN
      END


      SUBROUTINE AGCMP1(IAGE,NBYAGE,NSMPLE,NSTRAT,NST,PST,VBYAGE)
C-----------------------------------------------------------------------
C     "AGCMP1" FINDS AGE COMPOSITIONS FOR AN ENTIRE SEASON AND THE
C          VARIANCE OF THOSE COMPOSITIONS WHEN THE POPULATION SIZE IS
C          KNOWN. INPUTS ARE FROM A STRATIFIED RANDOM SAMPLING PROGRAM
C          FOR AGE COMPOSITION AND FROM AN ENUMERATING PROGRAM FOR A
C          SALMON POPULATION. INPUTS ARE PROPORTIONS BY AGE BY STRATUM
C          FROM SAMPLES, NUMBER SAMPLED BY STRATUM, NUMBER OF FISH
C          ENUMERATED BY STRATUM, NUMBER OF AGES, AND NUMBER OF STRATA.
C-----------------------------------------------------------------------
      REAL NBYAGE(10),VBYAGE(10),P(10),PST(10,15),NSMPLE(15),NSTRAT(15)
      REAL NTOT
      INTEGER HK
C-----------------------------------------------------------------------
C     IAGE           NUMBER OF AGE GROUPS - INPUT
C     I,HK           AGE,STRATUM - SUBSCRIPTS
C     NBYAGE(I)      ESTIMATED NUMBER BY AGE - OUTPUT
C     NSMPLE(HK)     NUMBER SAMPLED IN STRATA H - INPUT
C     NSTRAT(HK)     NUMBER IN POPULATION IN STRATA H - INPUT
C     NST            NUMBER OF STRATA - INPUT
```

```
C     NTOT           POPULATION SIZE
C     P(I)           ESTIMATED PROPORTION OF AGE I IN POPULATION
C     PST(I,HK)      PROPORTION OF AGE I IN SAMPLES TAKEN IN STRATA H -
C                         INPUT
C     VBYAGE(I)      ESTIMATED VARIANCE OF NUMBER BY AGE - OUTPUT
C-----------------------------------------------------------------------
C     FIND POPULATION SIZE
C-----------------------------------------------------------------------
      NTOT=0.
      DO  10  HK=1,NST
   10 NTOT=NSTRAT(HK)+NTOT
C-----------------------------------------------------------------------
C     FIND WEIGHTED PROPORTION FOR EACH AGE ACROSS STRATA
C-----------------------------------------------------------------------
      DO  30  I=1,IAGE
      P(I)=0.
      VBYAGE(I)=0.
      DO  20  HK=1,NST
      P(I)=NSTRAT(HK)/NTOT*PST(I,HK)+P(I)
C-----------------------------------------------------------------------
C     FIND VARIANCE OF NUMBERS BY AGE
C-----------------------------------------------------------------------
      T1=NSTRAT(HK)*NSTRAT(HK)*(NSTRAT(HK)-NSMPLE(HK))
      T2=PST(I,HK)*(1.-PST(I,HK))
      T3=(NSTRAT(HK)-1.)*(NSMPLE(HK)-1.)
   20 VBYAGE(I)=VBYAGE(I)+T1*T2/T3
C-----------------------------------------------------------------------
C     FIND NUMBERS BY AGE
C-----------------------------------------------------------------------
   30 NBYAGE(I)=P(I)*NTOT
      RETURN
      END


      SUBROUTINE AGCMP2(IAGE,NBYAGE,NSMPLE,NST,NSTRAT,NSTRTV,PST,VBYAGE)
C-----------------------------------------------------------------------
C     "AGCMP2" FINDS AGE COMPOSITIONS FOR AN ENTIRE SEASON AND THE
C          VARIANCE OF THOSE COMPOSITIONS WHEN THE POPULATION SIZE IS
C          ESTIMATED. INPUTS ARE FROM A STRATIFIED RANDOM SAMPLING
C          PROGRAM FOR AGE COMPOSITION AND FROM AN ENUMERATING PROGRAM
C          FOR A SALMON POPULATION. INPUTS ARE PROPORTIONS BY AGE BY
C          STRATUM IN SAMPLES, NUMBER SAMPLED BY STRATUM, ESTIMATED
C          NUMBER OF FISH IN PART OF THE POPULATION BY STRATUM AND THE
C          VARIANCE OF THE ESTIMATE, THE NUMBER OF AGES, AND THE NUMBER
C          OF STRATA.
C-----------------------------------------------------------------------
      REAL NBYAGE(10),VBYAGE(10),PST(10,15),NSMPLE(15),NSTRAT(15)
      REAL NSTRTV(15),NUMSTR
      INTEGER HK
```

```
C-----------------------------------------------------------------------
C     FA             FINITE POPULATION CORRECTION FACTOR FOR VARIANCE OF
C                         PST
C     IAGE           NUMBER OF AGE GROUPS - INPUT
C     I,HK           AGE,STRATUM - SUBSCRIPTS
C     NBYAGE(I)      ESTIMATED NUMBER BY AGE - OUTPUT
C     NSMPLE(HK)     NUMBER SAMPLED IN STRATUM - INPUT
C     NSTRAT(HK)     ESTIMATED NUMBER IN POPULATION IN STRATUM - INPUT
C     NSTRTV(HK)     VARIANCE FOR NSTRAT(HK) - INPUT
C     NST            NUMBER OF STRATA - INPUT
C     NUMSTR         NUMBER BY AGE BY STRATUM
C     PST(I,HK)      PROPORTION OF AGE I IN SAMPLES TAKEN IN STRATUM -
C                         INPUT
C     VBYAGE(I)      ESTIMATED VARIANCE OF NUMBER BY AGE - OUTPUT
C     VNSTRT         VARIANCE FOR NUMSTR
C     VPST           VARIANCE FOR PST
C-----------------------------------------------------------------------
C     FIND VARIANCE OF PROPORTION FOR STRATUM HK
C-----------------------------------------------------------------------
      DO  30  I=1,IAGE
      NBYAGE(I)=0.
      VBYAGE(I)=0.
      DO  30  HK=1,NST
      VPST=(1.-PST(I,HK))*PST(I,HK)/(NSMPLE(HK)-1.)
C-----------------------------------------------------------------------
C     FIND VARIANCE OF NUMBERS BY AGE BY STRATUM
C-----------------------------------------------------------------------
      FA=(NSTRAT(HK)-NSMPLE(HK))/(NSTRAT(HK)-1.)
      T1=NSTRAT(HK)*NSTRAT(HK)*VPST*FA
      T2=PST(I,HK)*PST(I,HK)*NSTRTV(HK)
      T3=NSTRTV(HK)*FA*VPST
      VNSTRT=T1+T2-T3
C-----------------------------------------------------------------------
C     ACCUMULATE VARIANCES OF NUMBERS BY AGE OVER STRATA
C-----------------------------------------------------------------------
      VBYAGE(I)=VBYAGE(I)+VNSTRT
C-----------------------------------------------------------------------
C     FIND NUMBERS BY AGE BY STRATUM HK
C-----------------------------------------------------------------------
      NUMSTR=NSTRAT(HK)*PST(I,HK)
C-----------------------------------------------------------------------
C     ACCUMULATE NUMBERS BY AGE OVER STRATA
C-----------------------------------------------------------------------
   30 NBYAGE(I)=NBYAGE(I)+NUMSTR
      RETURN
      END
```

APPENDIX B

Definitions of the notation used in this report. The term $V[\ ]$ is the variance of the variable in the brackets. Sampling strata are sampling periods.

B_{ij}	The relative bias in the allocation of catch of age i to river j
C	Number of fish in the catch
$\mathbf{C}$	A IxJ matrix of C_{ij}
c	A collected constant
C_{ij}	Catch of fish of age i bound for river j in a season
$\hat{C}_{ij}$	Estimated catch of fish of age i bound for river j in a season
$\hat{C}_i$	Estimated catch of fish of age i in a season
$\mathbf{C}_i$	An $Ix1$ vector of $C_{.i}$
$\mathbf{C}_j$	A $Jx1$ vector of $C_{.j}$
$C_{..k}$	Catch during sampling stratum k
$\hat{E}_{i.}$	Estimated escapement of fish of age i to all rivers in a season
$\hat{E}_{ij}$	Estimated escapement of fish of age i to river j in a season
$\hat{E}_{.jh}$	Estimated escapement of fish of all ages to river j during sampling stratum h
$\hat{E}_{ijh}$	Estimated escapement of fish of age i to river j during sampling stratum h
H_j	Number of strata in the sampling program to estimate the age composition of the season's escapement to river j
I	Number of ages
J	Number of rivers
K	Number of periods in the sampling program to estimate the age composition of the season's catch
N_{jh}	Number sampled during sampling stratum h in the sampling program for estimating the age composition of the escapement to river j

-Continued-

APPENDIX B (continued).

N_k	Number sampled during stratum k in the sampling program for estimating the age composition of the catch
P	A JxJ matrix of a subset of all P_{ij}
p_i	Proportion of the entire run (all rivers) of age i (sum of p_i over i equals one)
$\hat{P}_{ij}$	Estimated proportion of escapement to river j of age i
$\hat{P}_{ijh}$	Estimated proportion of escapement to river j of age i during sampling stratum h
$\hat{P}_{it}$	Estimated proportion of escapement to river t of age i
$\hat{Q}_i$	Estimated proportion of the catch that are age i
$\hat{Q}_{ik}$	Estimated proportion of the catch in sampling stratum k that are age i
Q	An $Ix1$ vector of a subset of all $\hat{Q}_i$.
R	Number of fish in the run
$R_{i\cdot}$	Number of fish in the run of age i
r_{ij}	Proportion of the fish of age i in the run represented by fish bound for river j (sum of r_{ij} over j equals one)
R_{ij}	Number of fish in the run of age i bound for river j
r_{it}	Proportion of the fish of age i in the run bound for river t
θ	An IxJ matrix of the fractions each run represent within an age group
U	A standard exploitation rate used as a scaling factor for determining relative bias in a catch allocation
U_i	Exploitation rate for fish of age i $(0 < U_i < 1)$
U_{ij}	Exploitation rate for fish of age i bound for river j $(0 < U_{ij} < 1)$
w_{ij}	Weighting factor that describes the difference between the actual exploitation rate U_{ij} as a linear function of a standard rate U
w_{it}	Weighting factor that describes the difference between the actual exploitation rate U_{it} as a linear function of a standard rate U

APPENDIX C

Worlund and Fredin (1962) propose to divide the catch in a mixed-stock fishery according to the frequency of a descriptive character, such as age, with the Difference method. A single allocation for a fishery with two attributes and two runs is made with Worlund and Fredin's Eq. 7:

$$F_a = \frac{R_a - P_{ba}}{P_{aa} - P_{ba}} \tag{C.1}$$

where F_a is the proportion of the catch from river a, R_a is the proportion of the with attribute a, P_{aa} is the proportion of the run to river a that has attribute a, and P_{ba} is the proportion of the run to river b that has attribute a. To solve the above equation, the Ps must be known or at least estimated. If the attribute is age, the Ps are part of the age composition of each run, and because the age composition of the escapement is the only estimate available for the age composition of the run, it is used to estimate the Ps. Worlund and Fredin's notation and the notation in this report are related as follows: $F_a=C_{.a}/C$; $R_a=C_{a.}/C$; $P_{aa}=E_{aa}/E_{.a}$; and $P_{ba}=E_{ab}/E_{.b}$.

The special case from Worlund and Fredin (1962) with two runs and two attributes can be expanded to a general case with J runs and I attributes (ages) to form Eq. 2.8 in the text. The Ps are arranged in the IxJ matrix P with each column the complete age composition of the run to a particular river:

$$\mathrm{P} = \begin{bmatrix} P_{aa} & P_{ba} \\ 1-P_{aa} & 1-P_{ba} \end{bmatrix} \tag{C.2}$$

In this example, $I=J=2$. For the general case:

$$\mathrm{P}^{-1}\mathrm{C}_i(1/C) = \mathrm{C}_j(1/C) \tag{C.3}$$

Because Eq. 7 in Worlund and Fredin (1962) produces proportions while the Difference method produces numbers, the factor $1/C$ is included in the above equation. The inverse of P is:

$$\mathrm{Q}^{-1} = \begin{bmatrix} \dfrac{1-P_{ba}}{P_{aa}-P_{ba}} & \dfrac{-P_{ba}}{P_{aa}-P_{ba}} \\[2ex] \dfrac{P_{aa}-1}{P_{aa}-P_{ba}} & \dfrac{P_{aa}}{P_{aa}-P_{ba}} \end{bmatrix} \tag{C.4}$$

Substituting the inverse into Eq. C.3 gives a solution:

$$\text{C.5)}\qquad \mathsf{F} = \begin{bmatrix} \dfrac{1-P_{ba}}{P_{aa}-P_{ba}} & \dfrac{-P_{ba}}{P_{aa}-P_{ba}} \\[2ex] \dfrac{P_{aa}-1}{P_{aa}-P_{ba}} & \dfrac{P_{aa}}{P_{aa}-P_{ba}} \end{bmatrix} \begin{bmatrix} R_a \\[2ex] 1-R_a \end{bmatrix} = \begin{bmatrix} \dfrac{R_a - P_{ba}}{P_{aa} - P_{ba}} \\[2ex] \dfrac{R_a - P_{aa}}{P_{ba} - P_{aa}} \end{bmatrix} = \begin{bmatrix} F_a \\[2ex] F_b \end{bmatrix}$$

The vector F has two elements, the first of which is the right-hand sides to Eq. 7 in Worlund and Fredin (1962); the second element is the complement of the first. Both elements F constitute the fractions in an allocation scheme.

APPENDIX D

The catch and escapement of sockeye salmon aged 4_2 and 5_2 to the River Styx and the River of No Return by day. The fishery and the runs which it uses are hypothetical.

	Catch				Escapement			
	Styx		No Return		Styx		No Return	
Date	4_2	5_2	4_2	5_2	4_2	5_2	4_2	5_2
6-1	0	0	0	0	0	0	0	0
6-2	0	0	0	0	0	0	0	0
6-3	0	0	0	0	0	0	0	0
6-4	0	0	0	0	0	0	0	0
6-5	0	0	0	0	0	0	0	0
6-6	0	0	0	0	0	0	0	15,700
6-7	0	0	0	0	0	0	0	15,700
6-8	0	0	0	16,515	0	0	0	16,000
6-9	0	0	0	19,681	0	0	0	16,000
6-10	0	0	0	22,319	0	0	0	19,600
6-11	0	4,142	4,970	23,955	0	0	0	25,000
6-12	0	0	0	0	0	0	0	29,000
6-13	0	0	0	0	0	0	0	14,785
6-14	0	0	0	0	0	0	0	17,619
6-15	0	6,618	7,942	35,996	0	0	0	19,981
6-16	0	8,442	10,130	35,996	0	0	0	21,445
6-17	0	9,793	11,751	35,996	0	0	0	49,000
6-18	0	10,569	12,683	34,443	0	0	4,450	51,000
6-19	0	0	0	0	0	3,708	9,420	53,300
6-20	0	0	0	0	0	7,850	9,600	17,304
6-21	0	0	0	0	0	8,000	9,600	17,304
6-22	48,787	19,033	22,840	28,977	0	8,000	3,818	17,304
6-23	49,720	19,810	23,772	24,316	0	3,182	4,870	16,557
6-24	49,720	20,704	24,844	22,529	0	4,058	5,649	49,000
6-25	60,907	20,704	24,844	19,422	0	4,707	6,097	45,400
6-26	0	0	0	0	0	5,081	22,380	42,300
6-27	0	0	0	0	0	18,650	25,380	8,323
6-28	0	0	0	0	0	21,150	27,240	6,984
6-29	115,909	19,033	22,840	12,197	0	22,700	6,560	6,471
6-30	131,446	17,635	21,162	12,197	0	5,467	6,828	5,578
7-1	141,080	16,431	19,717	0	0	5,690	7,136	19,600
7-2	152,266	14,489	17,386	0	62,800	5,946	7,136	16,000
7-3	0	0	0	0	14,013	5,946	31,980	16,000
7-4	0	0	0	0	14,280	26,650	31,980	3,503
7-5	0	0	0	0	14,280	26,650	30,600	3,503

-Continued-

APPENDIX D

The catch and escapement of sockeye salmon aged 4_2 and 5_2 to the River Styx and the River of No Return by day. The fishery and the runs which it uses are hypothetical (continued).

	Catch				Escapement			
	Styx		No Return		Styx		No Return	
Date	4_2	5_2	4_2	5_2	4_2	5_2	4_2	5_2
7-6	165,629	7,613	9,136	0	17,493	25,500	6,560	0
7-7	165,629	6,215	7,458	0	100,000	5,467	6,078	0
7-8	158,481	6,215	7,458	0	116,000	5,065	5,663	0
7-9	152,266	6,098	7,318	0	125,200	4,719	4,994	0
7-10	0	0	0	0	33,291	4,161	18,780	0
7-11	0	0	0	0	37,754	15,650	17,400	0
7-12	0	0	0	0	40,520	14,500	15,000	0
7-13	39,151	0	0	0	43,734	12,500	2,624	0
7-14	36,274	0	0	0	204,000	2,187	2,142	0
7-15	31,271	0	0	0	213,200	1,785	2,142	0
7-16	24,517	0	0	0	213,200	1,785	2,102	0
7-17	0	0	0	0	47,571	1,752	9,420	0
7-18	0	0	0	0	47,571	7,850	0	0
7-19	0	0	0	0	45,519	0	0	0
7-20	0	0	0	0	43,734	0	0	0
7-21	0	0	0	0	181,600	0	0	0
7-22	0	0	0	0	169,200	0	0	0
7-23	0	0	0	0	149,200	0	0	0
7-24	0	0	0	0	86,049	0	0	0
7-25	0	0	0	0	79,726	0	0	0
7-26	0	0	0	0	68,729	0	0	0
7-27	0	0	0	0	53,883	0	0	0
7-28	0	0	0	0	64,000	0	0	0
7-29	0	0	0	0	64,000	0	0	0
7-30	0	0	0	0	62,800	0	0	0
7-31	0	0	0	0	62,800	0	0	0
	C_{11}	C_{21}	C_{12}	C_{22}	E_{11}	E_{21}	E_{12}	E_{22}
	1,523,053	213,544	256,251	344,539	2,476,147	286,356	343,629	655,261
	1,736,597		600,790		2,762,503		998,890	
	2,337,387				3,761,393			
	6,098,780							